# Dynamic Launch Decisions for Satellite Constellation Operators

Washington State University

December 23, 2020

**Abstract**

Over the last decades, new technology has made low earth orbits (LEOs) more accessible, and the resulting increase in LEO satellites has increased the risk of collision. Orbital operations produce an externality through the creation of debris during launch, operation, and collisions which contributes to the risk of destruction. This effect is compounded as debris in orbit generates more debris through collisions with objects in orbit, possibly leading to a runaway effect called kessler syndrome. This paper develops a dynamic model of satellite operation incorporating two effects not considered in previous models: complementary network-like effects between satellites within the same operator's fleet (called a constellation) and collision avoidance efficiencies realized within constellations. The primary result is a preliminary model and the resulting analysis of the difference in satellite survival rates between constellations and and the societal fleet.

***Keywords:*** Orbits, Pollution, Economies of Scale, Externality

***JEL Codes:*** Q29, Q58, L25

**Acknowledgments:** I am the sole author and have received no contributions from others as of yet. This paper has been approved for dual submission in Econs 529 and Econs 594 by the instructors.

# Contents

# 1 Introduction

In September of 2019, the European Space Agency (ESA) released a tweet explaining that they had performed an maneuver to avoid a collision with a SpaceX Starlink Satellite in Low Earth Orbit (LEO).[1] While later reports[2] described it as the result of miscommunications, ESA used the opportunity to highlight the difficulties arising from coordinating avoidance maneuvers and how such coordination will become more difficult as the size and number of single purpose, single operator satellite fleets (satellite constellations) increase in low earth orbit.[3]

In spite of the fact that there is a lot of maneuvering room in outer space, the repeated interactions of periodic orbits make collisions probable. Consequently, objects in orbit are subject to both a congestion effect and a pollution effect. Congestion effects are primarily derived from avoiding collisions between artificial satellites. Pollution in orbit consists of debris, both natural and man-made, which increases the probability of an unforeseen collision. The defining feature of pollution in orbit is that it self-propagates as debris collides with itself and orbiting satellites to generate more debris. This dynamic underlies a key concern, originally explored by Kessler and Cour-Palais[4] that with sufficient mass in orbit (through satellite launches), the debris generating process could undergo a runaway effect rendering various orbital regions unusable. This cascade of collisions is often known as Kessler syndrome and may take place over various timescales.

Orbits may be divided into three primary groups, Low Earth Orbit (LEO), Medium Earth Orbit (MEO), and High Earth Orbit (HEO) where Geostationary Earth Orbit (GEO) considered a particular classification of HEO. While the topic of LEO allocation has historically remained somewhat unexplored, the last 6 years has seen a variety of new empirical studies

[1] ESA 2019a.
[2] Brodkin 2019.
[3] ESA 2019b.
[4] Kessler and Cour-Palais 1978.

and theoretical models published.

Macauley provided the first evidence of sub-optimal behavior in orbit by estimating the welfare loss due to the current method of assigning GEO slots to operators.[5] The potential losses due to anti-competitive behavior were highlighted by Adilov et al , who have analyzed the opportunities for strategic "warehousing" of non-functional satellites as a means of increasing competitive advantage by denying operating locations to competitors in GEO.[6]

The primary concern expressed in many of the published papers is whether or not orbits will be overused due to their common-pool nature, and which policies may prevent kessler syndrome. On this topic, Adilov, Alexander, and Cunningham examine pollution using a two-period salop model, incorporating the effects of launch debris on survival into the second period.[7] They find that the social planner generates debris and launches at lower rates than a free entry market.

This same result was found by Rao and Rondina in the context of an infinite period dynamic model. Their approach is defined by the assumption that there are numerous operators in a free entry environment who can each launch a single, identical constellation.[8] Rao, Burgess, and Kaffine use this model to estimate that achieving socially optimal behavior through orbital use fees could increase the value generated by the space industry by a factor of four.[9]

My objective is to explore the effects from organizing satellites into constellations on satellite launch decisions and operation. Although not explored in this paper, I hope to lay the groundwork for an analysis regarding pigouvian taxation as a solution to the externality of orbital debris. The primary results of this paper are: preliminary development of the extended dynamic model, characterization of the general solutions to both the constellation operators' problems and the fleet planner's problem, and an analysis of survival rates within

[5]Macauley 1998.

[6]Adilov, Cunningham, et al. 2019.

[7]Adilov, Alexander, and Cunningham 2015.

[8]Rao and Rondina 2020.

[9]Rao, Burgess, and Kaffine 2020.

constellations and the entire fleet.

This work is most closely related to Rao and Rondina's model[10] and the dynamic model developed by Adilov et all.[11] It is distinguished from the two models mentioned previously by accounting for collision avoidance efficiencies where satellites are less likely to collide with constellation members, as neither of the mentioned models accounts for this behavior. Additionally, it differs from Rao et al's model in that it allows constellations to be of different sizes. Adilov et al permit constellations, but assume that all constellation operators are profit maximizing firms. I explicitly provide a way to account for non-commercial space activities, such as military satellites. One key similarity of all three models is the form of the intertemporal laws of motion of both constellation sizes and debris. For debris, this involves accounting for existing debris, debris from launches, and debris from collisions. In the case of the fleet or constellation sizes, they all account for loss due to collisions and additions through launches.

The paper is organized as follows. In section 2 the underlying mathematical model is given for both constellation operators and a societal fleet planner. Section 3 examines how externalities generated by operating satellite constellations differ between constellation operators and fleet planners. It also examines various definitions of kessler syndrome and how that might be examined in this model. The paper concludes in section 4, with a discussion of outstanding issues, limitations to the model, and some areas of future interest. The appendix A contains mathematical derivations.

[10]Rao and Rondina 2020.

[11]Adilov, Alexander, and Cunningham 2018.

# 2 Model

This infinite period, dynamic model is an extension of Rao and Rondina's working paper[12] to include how operators deal with constellations. In summary, each constellation operator has a utility function and a loss function that depend on the number of satellites in the constellation, the total number of satellites in the societal fleet, and the amount of debris in orbit. The loss function describes the degradation and destruction of satellites within the constellation, and plays a critical role in the laws of motion of the satellite. The utility function is used to describe how increases in constellation size affect utility production, given the fleet size and debris levels.

## 2.1 Model Description

For a given set of orbits that interact regularly (an orbital "shell"), I assume there are $N$ operators, each of which has the potential to launch and operate a satellite constellation consisting of some endogenously chosen number of identical satellites.

Each constellation $i$ is described by the number of satellites in period $t$, where this satellite stock is denoted by $s_t^i$. Each operator of the constellation $i$ chooses the number of launches $x_t^i$ in each time period $t$. For simplicity, each launch is assumed to have a fixed cost $F$. In the aggregate, the satellite stock and launches for each period are represented by:

$$S_t = \sum_{i=1}^{N} s_t^i \tag{1}$$

$$X_t = \sum_{i=1}^{N} x_t^i \tag{2}$$

Satellites in a constellation are damaged or destroyed by collisions at the rate $l^i(s_t^i, S_t, D_t) \in$

[12] Rao and Rondina 2020.

$(0, 1)$. This includes collisions both within and without constellations. I assume that:

$$\frac{\partial l^i}{\partial D_t} > 0 \tag{3}$$

$$\frac{\partial l^i}{\partial S_t} > \frac{dl^i}{ds_t^i} = \frac{\partial l^i}{\partial s_t^i} + \frac{\partial l^i}{\partial S_t} > 0 \tag{4}$$

Equation 4 represents one of the key distinctions from previous dynamic models, in that the marginal risk of collision from adding a satellite to one's own constellation is lower than the marginal risk of collision from other operators adding satellites. The effects due to collision avoidance efficiencies within constellations will be examined in section 3. For any numerical examination, this assumption requires that:

$$0 > \frac{\partial l^i}{\partial s_t^i} > -\frac{\partial l^i}{\partial S_t} \tag{5}$$

This functional assumption, as described in eq. (4), is justified by the fact that when adding satellites to a constellation, an operator can choose to place the satellites in orbits that will have nearly zero probability of colliding with another satellite in the constellation. Operators who experience a collision between two of their own satellites experience a higher cost than if one satellite collides with the satellite of another operator, thus we would expect more care to be given to the internal organization of constellations. Consequent to this ex-ante optimal organization within constellations, the majority of collisions observed should occur between satellites of different constellations and not within the same constellation.

Between the launch rate and destruction rate, I obtain a law of motion for both constellation-level and society-level satellite stocks:

$$s_{t+1}^i = \left[1 - l^i(s_t^i, S_t, D_t)\right] s_t^i + x_t^i \tag{6}$$

$$S_{t+1} = X_t + \sum_{i=1}^{N} \left[1 - l^i(s_t^i, S_t, D_t)\right] s_t^i \tag{7}$$

where next period satellite stock equals the surviving satellite stock plus the total number of launches.

The level of debris in each period is represented by $D_t$, and is assumed to pose a latent risk. In particular, I assume that once debris is created, the risk it provides is only avoidable through not launching future satellites. In addition to naturally occurring debris, new debris is generated through the following three mechanisms.

- At launch, various processes can shed debris. Examples include leftover rocket stages, explosions during launch and deployment, and slag from solid rocket boosters.
- When destroyed, satellites will fragment and produce debris.
- Debris can collide with other debris, forming more but smaller debris.

This provides the following law of debris dynamics.

$$D_{t+1} = (1-\delta)D_t + mX_t + M \cdot \left( \sum_{i=1}^{N} l^i(s_t^i, S_t, D_t) \right) + g(D_t) \tag{8}$$

where $\delta$ represents the proportional decay of debris – through reentering the atmosphere – for a given shell, $M$ represents the debris generated from each collision, $m$ represents the debris generated from each launch, and $g(D_t)$ represents the new fragments from debris colliding with other debris. The parameters $\delta, M$, and $m$ are assumed to be exogenously determined and non-stochastic.

Satellite operators – whether commercial, governmental, research, or hobbyist[13] – expect to receive some utility from satellite operation. Because there are both firm and non-firm operators, we cannot denote this utility as exclusively profit utility nor consumption utility. Firms, such as television or internet providers experience this utility as profit, while government, research institutions, or hobbyists operating satellites will experience this utility as

[13]Notable examples of hobby satellites are the amateur (HAM) radio OSCAR satellites

consumption of the service provided. The choice of terminology acknowledges that the utility derived from orbit use is neither exclusively productive nor consumptive, and there may be interference between productive commercial and consumptive non-commercial operations.

Mathematically, this is represented by time-separable utility function:

$$u^i(s_t^i, S_t, D_t) \tag{9}$$

For simplicity, each constellation produces utility such that it is not affected by the size of any other given constellation. In the case that the constellation operator is a profit maximizing firm, this implies that they are a monopolist in their market. The period utility function may incorporate the effects of orbital congestion ($S_t$) or debris ($D_t$), accounting for their effect in producing value to the operator. Productive economies of scale within a constellation appear when $\frac{\partial^2 u^i}{\partial {s_t^i}^2} > 0$ for some values of $s_t^i, S_t, D_t$, and represents situations such as those of satellite-based internet providers that require a minimum number of satellites in the constellation to provide a given level of service.

## 2.2 Constellation Operator's Program

Often, in polluting environments, there is an ambient population that is harmed by pollution. Very rarely does satellite debris pose a hazard to those on earth, thus in this model the only population who's welfare is addressed are the satellite operators themselves. Each operator faces the following problem:

$$V^i(s_t^i, S_t, D_t) = \max_{x_t^i \geq 0} \quad u^i(s_t^i, S_t, D_t) - Fx_t^i + \beta V^i(s_{t+1}^i, S_{t+1}, D_{t+1}) \tag{10}$$

Subject To:

$$D_{t+1} = (1-\delta)D_t + mX_t + M \cdot \left( \sum_{i=1}^{N} s_t^i l^i(s_t^i, S_t, D_t) \right) + g(D_t) \tag{11}$$

$$s_{t+1}^i = \left[1 - l^i(s_t^i, S_t, D_t)\right] s_t^i + x_t^i \tag{12}$$

$$S_t = \sum_{i=1}^{N} s_t^i \tag{13}$$

$$X_t = \sum_{i=1}^{N} x_t^i \tag{14}$$

Where $V^i(\cdot)$ represents the value function for the constellation $i$ and $\beta$ represents a common discount factor across operators.

### 2.2.1 Characterizing solutions

These give the optimality condition:

$$\frac{F}{\beta} = \frac{\partial V^i}{\partial s_{t+1}^i} + \frac{\partial V^i}{\partial S_{t+1}} + m\frac{\partial V^i}{\partial D_{t+1}} \tag{15}$$

Iterating both forward and backward one condition gives the system

$$\frac{F}{\beta} = \frac{\partial V^i}{\partial s_t^i} + \frac{\partial V^i}{\partial S_t} + m\frac{\partial V^i}{\partial D_t} \tag{16}$$

$$\frac{F}{\beta} = \frac{\partial V^i}{\partial s_{t+1}^i} + \frac{\partial V^i}{\partial S_{t+1}} + m\frac{\partial V^i}{\partial D_{t+1}} \tag{17}$$

$$\frac{F}{\beta} = \frac{\partial V^i}{\partial s_{t+2}^i} + \frac{\partial V^i}{\partial S_{t+2}} + m\frac{\partial V^i}{\partial D_{t+2}} \tag{18}$$

The general envelope conditions are:

$$\frac{\partial V^i}{\partial s_t^i} - \frac{\partial u^i}{\partial s_t^i} = \beta\left[\frac{\partial V^i}{\partial s_{t+1}^i}\frac{\partial s_{t+1}^i}{\partial s_t^i} + \frac{\partial V^i}{\partial S_{t+1}}\frac{\partial S_{t+1}}{\partial s_t^i} + \frac{\partial V^i}{\partial D_{t+1}}\frac{\partial D_{t+1}}{\partial s_t^i}\right] \tag{19}$$

$$\frac{\partial V^i}{\partial S_t} - \frac{\partial u^i}{\partial S_t} = \beta\left[\frac{\partial V^i}{\partial s_{t+1}^i}\frac{\partial s_{t+1}^i}{\partial S_t} + \frac{\partial V^i}{\partial S_{t+1}}\frac{\partial S_{t+1}}{\partial S_t} + \frac{\partial V^i}{\partial D_{t+1}}\frac{\partial D_{t+1}}{\partial S_t}\right] \tag{20}$$

$$\frac{\partial V^i}{\partial D_t} - \frac{\partial u^i}{\partial D_t} = \beta\left[\frac{\partial V^i}{\partial s_{t+1}^i}\frac{\partial s_{t+1}^i}{\partial D_t} + \frac{\partial V^i}{\partial S_{t+1}}\frac{\partial S_{t+1}}{\partial D_t} + \frac{\partial V^i}{\partial D_{t+1}}\frac{\partial D_{t+1}}{\partial D_t}\right] \tag{21}$$

Note the linearity of the equations. This allows us to rewrite the system of envelope conditions as the following matrix expression.

$$\beta\begin{bmatrix} \frac{\partial s_{t+1}^i}{\partial s_t^i} & \frac{\partial S_{t+1}}{\partial s_t^i} & \frac{\partial D_{t+1}}{\partial s_t^i} \\ \frac{\partial s_{t+1}^i}{\partial S_t} & \frac{\partial S_{t+1}}{\partial S_t} & \frac{\partial D_{t+1}}{\partial S_t} \\ \frac{\partial s_{t+1}^i}{\partial D_t} & \frac{\partial S_{t+1}}{\partial D_t} & \frac{\partial D_{t+1}}{\partial D_t} \end{bmatrix}\begin{bmatrix} \frac{\partial V^i}{\partial s_{t+1}^i} \\ \frac{\partial V^i}{\partial S_{t+1}} \\ \frac{\partial V^i}{\partial D_{t+1}} \end{bmatrix} = \begin{bmatrix} \frac{\partial V^i}{\partial s_t^i} - \frac{\partial u^i}{\partial s_t^i} \\ \frac{\partial V^i}{\partial S_t} - \frac{\partial u^i}{\partial S_t} \\ \frac{\partial V^i}{\partial D_t} - \frac{\partial u^i}{\partial D_t} \end{bmatrix} \tag{22}$$

$$\beta A\nabla_{[s_{t+1}^i, S_{t+1}, D_{t+1}]}V^i = \nabla_{[s_t^i, S_t, D_t]}V^i - \nabla_{[s_t^i, S_t, D_t]}u^i \tag{23}$$

$$\beta A\nabla V_{t+1}^i = \nabla V_t^i - \nabla u_t^i \quad \text{for conciseness} \tag{24}$$

Solving for $\nabla V_{t+1}^i$, we get

$$\nabla V_{t+1}^i = (\beta A)^{-1}(\nabla V_t^i - \nabla u_t^i) \tag{25}$$

By iterating Eq. 25 one period, we get:

$$\beta A\nabla V_{t+2}^i = \nabla V_{t+1}^i - \nabla u_{t+1}^i \tag{26}$$

$$\nabla V_{t+2}^i = (\beta A)^{-1}\left((\beta A)^{-1}(\nabla V_t^i - \nabla u_t^i) - \nabla u_{t+1}^i\right) \tag{27}$$

With eqs. (25) and (27) substituted into the system of eqs. (16), (17) and (37), we can now solve for the optimal, functional form of $\nabla_{[s_t^i, S_t, D_t]}V^i$. Substituting this back into eq. (16) gives the euler equation for the optimal launch function $x_t^i(s_t^i, S_t, D_t)$.

### 2.2.2 Conditions for existence of a solution

For any given set of functional forms $l^i(\cdot), g(\cdot)$ and coefficients $m, M$, one must verify that $A$ is invertible for all values of the state and choice variables $s_t^i, S_t, D_t$, and $x_t^i$.

For the laws of motion eqs. (11) and (12), the matrix $A$ above is:

$$\begin{bmatrix} 1 - l^i(\cdot) - s_t^i \frac{\partial l^i}{\partial s_t^i} & 1 - l^i(\cdot) - s_t^i \frac{\partial l^i}{\partial s_t^i} - \sum_{j=1}^N s_t^j \frac{\partial l^j}{\partial S_t} & M\left[\frac{\partial l^i}{\partial s_t^i} + \sum_{j=1}^N \frac{\partial l^i}{\partial S_t}\right] \\ -s_t^i \frac{\partial l^i}{\partial S_t} & -\sum_{j=1}^N s_t^j \frac{\partial l^j}{\partial S_t} & M \sum_{j=1}^N \frac{\partial l^i}{\partial S_t} \\ -s_t^i \frac{\partial l^i}{\partial D_t} & -\sum_{j=1}^N s_t^j \frac{\partial l^j}{\partial D_t} & (1-\delta) + M\sum_{j=1}^N \frac{\partial l^i}{\partial D_t} + \frac{\partial g}{\partial D_t} \end{bmatrix} \tag{28}$$

## 2.3 Social Planner's Program

The social planner (or fleet planner to use Rao and Rondina's terminology), is tasked with maximizing the sum of the operators' benefits $W(\{s_t^i\}, S_t, D_t) = \sum_{i=1}^N V^i(s_t^i, S_t, D_t)$ as satellite debris rarely poses a threat to the welfare of those on earth.

$$W(\{s_t^i\}, D_t) = \max_{\{x_t^i\}_{i=1}^N \geq 0} \left(\sum_{i=1}^N u^i(s_t^i, S_t, D_t)\right) - FX_t + \beta W(\{s_{t+1}^i\}, D_{t+1}) \tag{29}$$

Subject To:

$$D_{t+1} = (1-\delta)D_t + mX_t + M \cdot \left(\sum_{i=1}^N s_t^i l^i(s_t^i, S_t, D_t)\right) + g(D_t) \tag{30}$$

$$s_{t+1}^i = \left[1 - l^i(s_t^i, S_t, D_t)\right] s_t^i + x_t^i \tag{31}$$

$$S_t = \sum_{i=1}^N s_t^i \tag{32}$$

$$X_t = \sum_{i=1}^N x_t^i \tag{33}$$

Solving for the euler equation follows the steps laid out in the section for constellation operators.

### 2.3.1 Characterizing solutions

The $N+1$ Envelope Conditions are:

$$\frac{\partial W}{\partial s_t^i} = \sum_{j=1}^{N} \frac{du^j}{ds_t^i} + \beta \left[ \sum_{j=1}^{N} \frac{\partial W}{\partial s_{t+1}^j} \frac{\partial s_{t+1}^j}{\partial s_t^i} + \frac{\partial W}{\partial D_{t+1}} \frac{\partial D_{t+1}}{\partial s_t^i} \right] \quad \forall i \in \{1, \ldots, N\} \tag{34}$$

$$\frac{\partial W}{\partial D_t} = \sum_{j=1}^{N} \frac{du^j}{dD_t} + \beta \left[ \sum_{j=1}^{N} \frac{\partial W}{\partial s_{t+1}^j} \frac{\partial s_{t+1}^j}{\partial D_t} + \frac{\partial W}{\partial D_{t+1}} \frac{\partial D_{t+1}}{\partial D_t} \right] \tag{35}$$

$$\nabla W_t - \sum_{j=1}^{N} \nabla u_t^j = \beta B \cdot \nabla W_{t+1} \tag{36}$$

Assuming $B$ is non-singular, we again find that:

$$\nabla W_{t+1} = (\beta B)^{-1} (\nabla W_t - \sum_{j=1}^{N} \nabla u_t^j) \tag{37}$$

The $N$ Optimality Conditions are:

$$0 = -F + \beta \left[ \sum_{j=1}^{N} \frac{\partial W}{\partial s_{t+1}^j} \frac{\partial s_{t+1}^j}{\partial x_t^i} + \frac{\partial W}{\partial D_{t+1}} \frac{\partial D_{t+1}}{\partial x_t^i} \right] \quad \forall i \in \{1, \ldots, N\} \tag{38}$$

$$\frac{F}{\beta} \vec{1} = C \nabla W_{t+1} \tag{39}$$

Where $C$ is a $N \times N+1$ matrix. Iterating eq. (38) one period forward (from $t+1$ to $t+2$) for $i = 1$ and and substituting in eq. (37) twice provides the final equation for a system of $N+1$ equations for $\nabla W_t$.

Finally, iterating eq. (38) one period backward (from $t+1$ to $t$) for all $i$, and substituting the previously found values for $\nabla W_t$ into these optimality conditions defines the system of euler equations that characterize $\{x_t^i\}$.

# 3 Analysis

## 3.1 Survival Ratios

In line with basic theories of common-pool resources, we expect there to be a negative externality incurred on other constellations when a constellation increases their own satellite stock (resource usage). This externality comes from two effects, congestion and pollution. Congestion, due to size of the societal fleet, may affect the utility achieved by other satellite operators and it increases the probability of a satellite on satellite collision. Pollution, the debris in all future periods, increase the rate of degradation and destruction of satellites. When a constellation operator increases their satellite stock, the other operators experience a loss of welfare through both congestion and pollution. One way to measure the effects of satellite operations is through survival rates.

The survival rate for a constellation $i$ is defined as $R_i = 1 - l^i(\cdot)$, the proportion of satellites that were not lost (degraded nor destroyed) between period $t$ and $t+1$. Thus the marginal survival rate represents the additional loss of satellites due to a slightly larger constellation or fleet stock.

Mathematically the survival rates for a constellation and for society's fleet are defined as:

$$R_i = \frac{s^i_{t+1} - x^i_t}{s^i_t} = 1 - l^i(s^i_t, S_t, D_t) \tag{40}$$

$$R = \frac{S_{t+1} - X_t}{S_t} = \frac{\sum_{i=1}^{N} s^i_t[1 - l^i(s^i_t, S_t, D_t)]}{S_t} \tag{41}$$

In this case, the fleet survival rate eq. (41), represents the proportion of satellites in period $t+1$ that survived from period $t$.

The marginal survival rates when a given constellation $i$ changes size are:

$$\frac{\partial R_i}{\partial s_t^i} = -\left(\frac{\partial l^i}{\partial s_t^i} + \frac{\partial l^i}{\partial S_t}\frac{\partial S_t}{\partial s_t^i}\right) = -\frac{\partial l^i}{\partial s_t^i} - \frac{\partial l^i}{\partial S_t} \tag{42}$$

$$\frac{\partial R}{\partial s_t^i} = \frac{S_t \sum_{i=1}^N \left([1 - l^i(s_t^i, S_t, D_t)] + s_t^i[-\frac{\partial l^i}{\partial s_t^i} - \frac{\partial l^i}{\partial S_t}\frac{\partial S_t}{\partial s_t^i}]\right) - \left(\sum_{i=1}^N s_t^i[1 - l^i(s_t^i, S_t, D_t)]\right)}{(S_t)^2} \tag{43}$$

$$= \sum_{i=1}^N \left[\frac{R_i}{S_t}\right] - \frac{R}{S_t} + \sum_{i=1}^N \frac{s_t^i}{S_t}\frac{\partial R_i}{\partial s_t^i} \tag{44}$$

Note that $\sum_{i=1}^N \frac{s_t^i}{S_t}\frac{\partial R_i}{\partial s_t^i}$ is the weighted, average marginal survival rate across constellation operators. The derivation of eq. (44) is in Appendix A.1. Direct comparison between the marginal survival rates of an individual operator and the social planner's fleet cannot proceed further without specifying the functional loss forms $l^i(\cdot)$ and specifying which firm will be compared to society. In spite of this, conditions on the average effects can be developed as follows.

The marginal survival rate of the fleet is greater than the weighted, arithmetic mean of marginal survival rates of the constellations when:

$$\sum_{i=1}^N \left[\frac{R_i}{S_t}\right] - \frac{R}{S_t} + \sum_{i=1}^N \frac{s_t^i}{S_t}\frac{\partial R_i}{\partial s_t^i} \leq \sum_{i=1}^N \frac{s_t^i}{S_t}\frac{\partial R_i}{\partial s_t^i} \tag{45}$$

$$\sum_{i=1}^N R_i - R \leq 0 \tag{46}$$

$$\sum_{i=1}^N [1 - l^i(s_t^i, S_t, D_t)] - \sum_{i=1}^N s_t^i[1 - l^i(s_t^i, S_t, D_t)] \leq 0 \tag{47}$$

$$\sum_{i=1}^N (1 - s_t^i)[1 - l^i(s_t^i, S_t, D_t)] \leq 0 \tag{48}$$

which is true if every constellation has at least one satellite. As any constellation of interest has at least one satellite and $\frac{\partial R_i}{\partial s_t^i} < 0$ from the assumption on collision mechanics that $\frac{dl^i}{ds_t^i} > 0$, we conclude that the marginal survival rate of the entire satellite fleet is lower than the weighted arithmetic mean of marginal survival rates across constellations. Note that it is possible for some constellations to have a lower marginal survival rate than the fleet, but the survival rate for many operators must be higher than the societal rate. Consequently, we would expect many operators to underestimate the impact of their behaviors on others if they use their own observed or expected risk factors to estimate the risk they impose on others.

## 3.2 Kessler Syndrome

Rao and Rondina[14] interpret their model in terms of a physical kessler syndrome, while Adilov et al[15] develop the concept of an economic kessler syndrome. Generalizing Rao's approach, I define the kessler region as the set of states such that the debris stock will tend to infinity, and kessler syndrome as when the state is in the kessler region. Formally, the kessler region is:

$$\vartheta_1 = \left\{ (\{s_t^i\}, D_t) : X_t(\{s_t^i\}, D_t) \wedge (\{s_t^i\}, D_t) \Rightarrow \lim_{t \to \infty} D_{t+1} = \infty \right\} \tag{49}$$

I suspect, but have not been able to prove, that an equivalent condition is:

$$\vartheta_2 = \left\{ (\{s_t^i\}, D_t) : X_t(\{s_t^i\}, D_t) \wedge (\{s_t^i\}, D_t) \Rightarrow \frac{\partial (D_{t+1} - D_t)}{\partial D_t} > 0 \right\} \tag{50}$$

If the assumption holds, then a condition for a physical kessler region in this model is:

$$\vartheta_2 = \left\{ (\{s_t^i\}, D_t) : X_t(\{s_t^i\}, D_t) \wedge (\{s_t^i\}, D_t) \Rightarrow m \frac{\partial X_t(\{s_t^i\}, D_t)}{\partial D_t} + M \cdot \left( \sum_{i=1}^{N} \frac{\partial l^i}{\partial D_t} \right) + g(D_t) > \delta \right\} \tag{51}$$

[14] Rao and Rondina 2020.

[15] Adilov, Alexander, and Cunningham 2018.

Adilov et al[16] define an economic kessler syndrome (and thus kessler region) along the lines of

$$\vartheta_3 = \left\{(\{s_t^i\}, D_t) : X_t(\{s_t^i\}, D_t) = 0\right\} \tag{52}$$

This represents the conditions under which adding satellites to the orbit becomes unprofitable. They are able to establish conditions under which an economic kessler syndrome precedes a physical kessler syndrome. Some modification of the conditions are required to get them to match the terminology in this model, but I have not yet completed that work. The benefit of this definition is that the euler equation defining $X_t(\cdot)$ can be searched for the states that imply $X_t = 0, \forall t$ [17].

# 4 Summary and Concluding Remarks

Although significant work remains to describe the impacts of organizing satellites as constellations, I have been able to achieve many of preliminary milestones. Foremost among these is the section which characterizes the general euler equation and provides a simple set of conditions for existence. This has opened a possible numerical approach to determining the economic kessler region. Additionally, we have identified some preliminary results constraining the fleet's marginal survival rate to be less than the weighted arithmetic average of the constellations' marginal survival rate. This result – consistent with the assumptions on avoidance efficiencies – highlights the nature of the externality imposed by operating and launching satellites.

There are three primary limitations within the model. The first is the implicit assumption on $u(\cdot)$ that firms operating constellations act monopolistically, i.e. they do not compete in

[16] Adilov, Alexander, and Cunningham 2018.

[17] I have yet to conduct such a search, but plan on doing so as part of a numerical simulation.

the same market. This is an unreasonable assumption as there are already firms attempting to compete in LEO as satellite internet providers, most notably SpaceX's Starlink and OneWeb. The second primary limitation is that of computational difficulty, due to the large state space of the model. Even the simple constellation operator's problem presented here requires intensive algebra to define the euler equation. The typical response to this issue is to use computational methods to estimate the value and policy functions for both the operators and the fleet planner, but this has the disadvantage of reducing generalizability. The third limitation is that the model doesn't track individual satellites through their lifetime, particularly the decision to deorbit or park the satellite. Thus I ignore satellite both ex-ante and ex-post heterogeneity, preventing the analysis of how policies affect satellite disposal decisions.

The ultimate goal of developing this model is to facilitate policy analyses geared towards optimizing the productive use of orbits. As previous work has suggested that taxation may be an appropriate policy response to encourage optimal use, I hope to be able to address the following questions with this model, at least in specific (computational) cases:

1. Do avoidance efficiencies affect the optimal tax schedule for a given constellation operator? E.g. one constellation may be able to almost completely eliminate the chance of a within constellation collision, while another may not. Should they be taxed at different rates?

2. Do productive economies of scale require a non-linear tax schedule to optimize orbit use?

3. How does the decay rate $\delta$ (which depends on constellation altitude) affect the optimal tax schedule?

One concern, tangential to work by Adilov, et al[18] is that there may be ways for firms

[18] Adilov, Cunningham, et al. 2019.

to increase barriers to entry for competitors by holding more satellites in orbit. If this is the case, it begs the question of whether this will move the satellite stock closer to kessler syndrome through an increase in the fleet stock of satellites, or if the avoidance efficiencies are sufficient to move it farther from kessler syndrome. This is a crucial question to answer as it could inform policies regarding launch quotas and taxation.

Finally, a glaring issue is that the model is deterministic, and thus doesn't include risk aversion. The variety of satellite operators that currently exist include militaries operating intelligence and communications satellites. One would expect that the critical nature of these constellations would imply a high level of risk aversion in these operators, making this an important area of study.

# References

ESA (Sept. 2, 2019a). *For the first time ever, ESA has performed a 'collision avoidance manoeuvre' to protect one of its satellites from colliding with a 'mega constellation' #SpaceTraffic*. https://twitter.com/esaoperations.

Brodkin, Jon (Sept. 3, 2019). *SpaceX satellite was on "collision course" until ESA satellite was re-routed*. English. Statement from SpaceX to ARS Technica. Ars Technica. URL: https://arstechnica.com/information-technology/2019/09/spacex-satellite-was-on-collision-course-until-esa-satellite-was-re-routed/.

ESA (Sept. 3, 2019b). *ESA spacecraft dodges large constellation*. English. European Space Agency. URL: http://www.esa.int/Safety_Security/ESA_spacecraft_dodges_large_constellation.

Kessler, Donald J. and Burton G. Cour-Palais (1978). "Collision frequency of artificial satellites: The creation of a debris belt". In: *Journal of Geophysical Research: Space Physics* 83.A6, pp. 2637–2646. DOI: 10.1029/JA083iA06p02637. eprint: https://agupubs.onlinelibrary.wiley.com/doi/pdf/10.1029/JA083iA06p02637. URL: https://agupubs.onlinelibrary.wiley.com/doi/abs/10.1029/JA083iA06p02637.

Macauley, Molly K (1998). "Allocation of Orbit and Spectrum Resources for Regional Communications: What's At Stake?" In: *The Journal of Law and Economics* 41.S2, pp. 737–764. ISSN: 0022-2186. DOI: 10.1086/467411.

Adilov, Cunningham, et al. (Apr. 2019). "LEFT FOR DEAD: ANTI-COMPETITIVE BEHAVIOR IN ORBITAL SPACE". In: *Economic Inquiry* 57. DOI: 10.1111/ecin.12790.

Adilov, Alexander, and Cunningham (2015). "An Economic Analysis of Earth Orbit Pollution". In: *Environmental and Resource Economics* 60.1, pp. 81–98. ISSN: 0924-6460. DOI: 10.1007/s10640-013-9758-4.

Rao and Rondina (Feb. 2020). *Cost in Space:Debris and Collision Risk in the Orbital Commons*. Working Paper. Middlebury College — UC San Diego. NA.

Rao, Burgess, and Kaffine (2020). "Orbital-use fees could more than quadruple the value of the space industry". In: *Proceedings of the National Academy of Sciences* 117.23, pp. 12756–12762. ISSN: 0027-8424. DOI: 10.1073/pnas.1921260117. eprint: https://www.pnas.org/content/117/23/12756.full.pdf. URL: https://www.pnas.org/content/117/23/12756.

Adilov, Alexander, and Cunningham (2018). "An economic "Kessler Syndrome": A dynamic model of earth orbit debris". In: *Economics Letters* 166, pp. 79–82. ISSN: 0165-1765. DOI: 10.1016/j.econlet.2018.02.025.

# A Derivations

## A.1 Survival Rates

$$\frac{\partial R_i}{\partial s_t^i} = -\left(\frac{\partial l^i}{\partial s_t^i} + \frac{\partial l^i}{\partial S_t}\frac{\partial S_t}{\partial s_t^i}\right) = -\frac{\partial l^i}{\partial s_t^i} - \frac{\partial l^i}{\partial S_t} \tag{53}$$

$$\frac{\partial R}{\partial s_t^i} = \frac{S_t \sum_{i=1}^N \left([1 - l^i(s_t^i, S_t, D_t)] + s_t^i[-\frac{\partial l^i}{\partial s_t^i} - \frac{\partial l^i}{\partial S_t}\frac{\partial S_t}{\partial s_t^i}]\right) - \left(\sum_{i=1}^N s_t^i[1 - l^i(s_t^i, S_t, D_t)]\right)}{(S_t)^2} \tag{54}$$

$$= \sum_{i=1}^N \left[\frac{S_t[1 - l^i(s_t^i, S_t, D_t)]}{(S_t)^2} - \frac{s_t^i[1 - l^i(s_t^i, S_t, D_t)]}{(S_t)^2}\right] + \sum_{i=1}^N \frac{s_t^i S_t[-\frac{\partial l^i}{\partial s_t^i} - \frac{\partial l^i}{\partial S_t}]}{(S_t)^2} \tag{55}$$

$$= \sum_{i=1}^N \left[\frac{S_t - s_t^i}{(S_t)^2}[1 - l^i(s_t^i, S_t, D_t)]\right] + \sum_{i=1}^N \frac{s_t^i}{S_t}\frac{\partial R_i}{\partial s_t^i} \tag{56}$$

$$= \sum_{i=1}^N \left[\frac{1}{S_t}[1 - l^i(s_t^i, S_t, D_t)]\right] - \frac{R}{S_t} + \sum_{i=1}^N \frac{s_t^i}{S_t}\frac{\partial R_i}{\partial s_t^i} \tag{57}$$

$$= \sum_{i=1}^N \left[\frac{R_i}{S_t}\right] - \frac{R}{S_t} + \sum_{i=1}^N \frac{s_t^i}{S_t}\frac{\partial R_i}{\partial s_t^i} \tag{58}$$