Dynamic Launch Decisions for Satellite Constellation Operators

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Abstract

Over the last decades, new technology has make low earth orbits (LEOs) more accessible, and the resulting increase in LEO satellites has increased the risk of collision. Because debris in orbit generates more debris through collisions with objects in orbit and the debris created during launch and operation imposes a negative externality on other operators, optimal use of orbits is believed to not occur under free entry. This paper develops a dynamic model of satellite operation incorporating two effects not considered in previous models. The first effect is complementarity between satellites within the same operator's fleet (called a constellation). The second effect is collision avoidance efficiencies that exist within constellations. The primary result is a theoretical model and the resulting analysis of the difference in survival rates between constellation operators and society.

Keywords: Orbits, Pollution, Economies of Scale, Externality

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Contents

1 Introduction

In September of 2019, the European Space Agency (ESA) released a tweet explaining that they had performed an maneuver to avoid a collision with a SpaceX Starlink Satellite in Low Earth Orbit (LEO)[1]. While later reports[2] described it as the result of miscommunications, ESA used the opportunity to highlight the difficulties arising from coordinating avoidance maneuvers and how such coordination will become more difficult as the size and number of single purpose, single operator satellite fleets (satellite constellations) increase in low earth orbit[3].

In spite of the fact that there is a lot of maneuvering room in outer space, the repeated interactions of periodic orbits make collisions probable. Consequently, objects in orbit are subject to both a congestion effect and a pollution effect. Congestion effects are primarily derived from avoiding collisions between artificial satellites. Pollution in orbit consists of debris, both natural and man-made, which increases the probability of an unforeseen collision. The defining dynamic of pollution in orbit is that it self-propagates as debris collides with itself and orbiting satellites to generate more debris. This dynamic underlies a key concern, originally explored by Kessler and Cour-Palais [4] that with sufficient mass in orbit (through satellite launches), the debris generating process could undergo a runaway effect rendering various orbital regions unusable. This cascade of collisions is often known as Kessler syndrome and may take place over various timescales.

Orbits may be divided into three primary groups, Low Earth Orbit (LEO, less than 2,400km in altitude[5]), Medium Earth Orbit (MEO), and High Earth Orbit (HEO) with Geostationary Earth Orbit (GEO) considered a particular classification of orbit. While the topic of LEO allocation has historically remained somewhat unexplored, the last 6 years has seen a variety of new empirical studies and theoretical models published. In general, three primary, related topics appear in the literature: Allocative Efficiency, Policy Intervention, and the occurrence of Kessler Syndrome.

The primary concern is to establish whether or not orbits will be overused due to their common-pool nature, and if allocation procedures are efficient. The earliest theoretical model I have found, due to Adilov, Alexander, and Cunningham [6], examines pollution using a two-period salop model, incorporating the effects of launch debris on survival into the second period. They find that the social planner generates debris and launches at lower rates than a free entry market. This same result was found by Rao and Rondina [7] in the context of an infinite period dynamic model. They approach the problem in the case where numerous operators in a free entry environment can each launch a single, identical satellite.

In addition to analyzing the allocative results, a significant area of interest is what impact various policy interventions can have. The policies analyzed and methods used have been widely varied. Macauley [8] provided the first evidence of sub-optimal behavior in orbit by estimating the welfare lose due to the current method of assigning GEO slots to operators. The potential losses due to anti-competitive behavior was highlighted by Adilov et al [9], who have analyzed the opportunities for strategic "warehousing" of non-functional satellites as a means of increasing competitive advantage by denying operating locations to competitors in

GEO. Grzelka and Wagner [10] explore methods of encouraging satellite quality (in terms of debris) and cleanup. Finally, Rao and Rondina [11] estimate that achieving socially optimal behavior through orbital use fees could increase the value generated by the space industry by a factor of four.

Although Kessler and Cour-Palais determined that a runaway pollution effect could make a set of orbits physically unusable, Adilov et al [12] have shown that economic benefits provided by orbits will drop sufficiently to make the net marginal benefit of new launches negative before the physical kessler syndrome occurs.

This paper's objective is to lay the foundations necessary to explore the effects of organizing satellites as constellations , particularly through collision avoidance efficiencies and economies of scale in utility production. No model as of yet has examined these aspects of orbit use. The primary analytical result aside from developing the preliminary model and characterizing general solutions is to examine if there exists a negative externality related to survival rates.

The paper is organized as follows. Section 2 describes the mathematical organization of the model for the cases of independent constellation operators and a social planner operating the same constellations. Section 3 evaluates the differences between the constellation operators and social planner models, particularly the difference between marginal survival rates . Section 4 concludes with a discussion of potential extensions and topics which have not yet been addressed.

2 Model

The dynamic model is an extension of Rao and Rondina's working paper [7] (specifically their non-stochastic model) to include how operators deal with constellations.

2.1 Model Outline

For a given orbital shell (a set of orbits that interact regularly), I assume there are N operators, each of which has the potential to launch and operate a satellite constellation consisting of some endogenously chosen number of identical satellites.

Each constellation operator has a personal satellite stock s_t^i in each period, and chooses the number of launches in that time period x_t^i. For simplicity, each launch is assumed to have a fixed cost F. In the aggregate, the satellite stock and launches for each period are represented by:

$$S_t = \sum_{i=1}^{N} s_t^i \tag{1}$$

$$X_t = \sum_{i=1}^{N} x_t^i \tag{2}$$

Satellites in a constellation are damaged or destroyed at the rate $l^i(s^i_t, S_t, D_t)$, which is assumed to be increasing in s^i_t, S_t, and D_t (debris, see below). One key difference from the previous models of Rao and Rondina [7] and Adilov et al [12] is that this model allows the rate of collision within constellations and between constellations to be different. This reflects the assumption that an operator can and will put more effort into protecting the satellites within the constellation from each other. One example of how this can be accomplished is that while choosing the orbits for a constellation, it is possible for an operator to chose a set of trajectories that best meet their needs and minimizes the risk of collision within the constellation. Mathematically this is represented by the inclusion of s^i_t in l^i. Together with the launch rate, we obtain a law of motion for both constellation-level and society-level satellite stocks.

$$s^i_{t+1} = \left[1 - l^i(s^i_t, S_t, D_t)\right] s^i_t + x^i_t \tag{3}$$

$$S_{t+1} = X_t + \sum_{i=1}^{N} \left[1 - l^i(s^i_t, S_t, D_t)\right] s^i_t \tag{4}$$

The level of debris in each period is represented by D_t, and is assumed to pose a latent risk. In particular, it is assumed that once debris is created, the risk it provides is only avoidable through not launching future satellites. In addition to naturally occurring debris, debris is generated through the following three mechanisms.

- At launch, various processes can shed debris. Examples include leftover rocket stages, explosions during launch and deployment, and slag from solid rocket boosters.
- When destroyed, satellites will fragment and produce debris.
- Debris can collide with other debris, forming more but smaller debris.

This provides the following law of debris dynamics.

$$D_{t+1} = (1-\delta)D_t + mX_t + M \cdot \left(\sum_{i=1}^{N} l^i(s^i_t, S_t, D_t)\right) + g(D_t) \tag{5}$$

where δ represents the proportional decay of debris – through reentering the atmosphere – for a given shell, M represents the debris generated from each collision, m represents the debris generated from each launch, and $g(D_t)$ represents the new fragments from debris colliding with other debris.

Each constellation $i \in 1, \ldots, N$ produces value for their operator at each period according to the function:

$$u^i(s^i_t, S_t, D_t) \tag{6}$$

Productive economies of scale within a constellation appear when $\frac{\partial^2 u^i}{\partial {s^i_t}^2} > 0$ for some values of s^i_t, S_t, D_t. Of note is that firms are assumed to produce value monopolistically, i.e. there are no substitution nor complementary effects between constellations. This allows us to examine the effects of economies of scale and collision avoidance efficiencies without incorporating the effects of competition. The period value function may incorporate the effects of orbit and congestion debris, accounting for their effect in producing value to the operator. Adilov et al analyzed the effects of competition between operators in launch decisions [9]. A similar approach could be used, but would add significant complexity to the model.

One key note is the choice of the word "value" as opposed to "profit". Historically, space operations have been motivated by objectives other than profit, such as national security, scientific inquisitiveness, to enhance hobbies such as amature radio, or to quote President John F. Kennedy, "...because [it] is hard." [13]. This choice of terminology acknowledges that orbit use is not exclusively commercial and there may be interference between commercial and non-commercial operations.

2.2 Constellation Operator's Program

Often, in polluting environments, there is an ambient population that is harmed by pollution. Very rarely does satellite debris pose a hazard to those on earth, thus in this model the only population who's welfare is addressed are the satellite operators themselves. Each operator faces the following problem:

$$V^i(s^i_t, S_t, D_t) = \max_{x^i_t \geq 0} \quad u^i(s^i_t, S_t, D_t) - F x^i_t + \beta V^i(s^i_{t+1}, S_{t+1}, D_{t+1}) \tag{7}$$

Subject To:

$$D_{t+1} = (1-\delta)D_t + mX_t + M \cdot \left(\sum_{i=1}^{N} s^i_t l^i(s^i_t, S_t, D_t) \right) + g(D_t) \tag{8}$$

$$s^i_{t+1} = \left[1 - l^i(s^i_t, S_t, D_t)\right] s^i_t + x^i_t \tag{9}$$

$$S_t = \sum_{i=1}^{N} s^i_t \tag{10}$$

$$X_t = \sum_{i=1}^{N} x^i_t \tag{11}$$

Giving the optimality condition:

$$\frac{F}{\beta} = \frac{\partial V^i}{\partial s^i_{t+1}} + \frac{\partial V^i}{\partial S_{t+1}} + m \frac{\partial V^i}{\partial D_{t+1}} \tag{12}$$

Iterating both forward and backward one condition gives the system

$$\frac{F}{\beta} = \frac{\partial V^i}{\partial s_t^i} + \frac{\partial V^i}{\partial S_t} + m\frac{\partial V^i}{\partial D_t} \tag{13}$$

$$\frac{F}{\beta} = \frac{\partial V^i}{\partial s_{t+1}^i} + \frac{\partial V^i}{\partial S_{t+1}} + m\frac{\partial V^i}{\partial D_{t+1}} \tag{14}$$

$$\frac{F}{\beta} = \frac{\partial V^i}{\partial s_{t+2}^i} + \frac{\partial V^i}{\partial S_{t+2}} + m\frac{\partial V^i}{\partial D_{t+2}} \tag{15}$$

The general envelope conditions are:

$$\frac{\partial V^i}{\partial s_t^i} - \frac{\partial u^i}{\partial s_t^i} = \beta\left[\frac{\partial V^i}{\partial s_{t+1}^i}\frac{\partial s_{t+1}^i}{\partial s_t^i} + \frac{\partial V^i}{\partial S_{t+1}}\frac{\partial S_{t+1}}{\partial s_t^i} + \frac{\partial V^i}{\partial D_{t+1}}\frac{\partial D_{t+1}}{\partial s_t^i}\right] \tag{16}$$

$$\frac{\partial V^i}{\partial S_t} - \frac{\partial u^i}{\partial S_t} = \beta\left[\frac{\partial V^i}{\partial s_{t+1}^i}\frac{\partial s_{t+1}^i}{\partial S_t} + \frac{\partial V^i}{\partial S_{t+1}}\frac{\partial S_{t+1}}{\partial S_t} + \frac{\partial V^i}{\partial D_{t+1}}\frac{\partial D_{t+1}}{\partial S_t}\right] \tag{17}$$

$$\frac{\partial V^i}{\partial D_t} - \frac{\partial u^i}{\partial D_t} = \beta\left[\frac{\partial V^i}{\partial s_{t+1}^i}\frac{\partial s_{t+1}^i}{\partial D_t} + \frac{\partial V^i}{\partial S_{t+1}}\frac{\partial S_{t+1}}{\partial D_t} + \frac{\partial V^i}{\partial D_{t+1}}\frac{\partial D_{t+1}}{\partial D_t}\right] \tag{18}$$

Note the linearity of the equations. This allows us to rewrite the system of envelope conditions as the following matrix expression.

$$\beta\begin{bmatrix}\frac{\partial s_{t+1}^i}{\partial s_t^i} & \frac{\partial S_{t+1}}{\partial s_t^i} & \frac{\partial D_{t+1}}{\partial s_t^i}\\ \frac{\partial s_{t+1}^i}{\partial S_t} & \frac{\partial S_{t+1}}{\partial S_t} & \frac{\partial D_{t+1}}{\partial S_t}\\ \frac{\partial s_{t+1}^i}{\partial D_t} & \frac{\partial S_{t+1}}{\partial D_t} & \frac{\partial D_{t+1}}{\partial D_t}\end{bmatrix}\begin{bmatrix}\frac{\partial V^i}{\partial s_{t+1}^i}\\ \frac{\partial V^i}{\partial S_{t+1}}\\ \frac{\partial V^i}{\partial D_{t+1}}\end{bmatrix} = \begin{bmatrix}\frac{\partial V^i}{\partial s_t^i} - \frac{\partial u^i}{\partial s_t^i}\\ \frac{\partial V^i}{\partial S_t} - \frac{\partial u^i}{\partial S_t}\\ \frac{\partial V^i}{\partial D_t} - \frac{\partial u^i}{\partial D_t}\end{bmatrix} \tag{19}$$

$$\beta A\nabla_{[s_{t+1}^i, S_{t+1}, D_{t+1}]}V^i = \nabla_{[s_t^i, S_t, D_t]}V^i - \nabla_{[s_t^i, S_t, D_t]}u^i \tag{20}$$

$$\beta A\nabla V_{t+1}^i = \nabla V_t^i - \nabla u_t^i \text{ for conciseness} \tag{21}$$

Solving for ∇V_{t+1}^i, we get

$$\nabla V_{t+1}^i = (\beta A)^{-1}(\nabla V_t^i - \nabla u_t^i) \tag{22}$$

By iterating Eq. 22 one period, we get:

$$\beta A\nabla V_{t+2}^i = \nabla V_{t+1}^i - \nabla u_{t+1}^i \tag{23}$$

$$\nabla V_{t+2}^i = (\beta A)^{-1}\left((\beta A)^{-1}(\nabla V_t^i - \nabla u_t^i) - \nabla u_{t+1}^i\right) \tag{24}$$

With eqs. (22) and (24) substituted into the system of eqs. (13), (14) and (34), we can now solve for the optimal, functional form of $\nabla_{[s_t^i, S_t, D_t]}V^i$. Substituting this back into eq. (13) gives the euler equation for the optimal launch function $x_t^i(s_t^i, S_t, D_t)$.

2.2.1 Note

For any given set of functional forms $l^i(\cdot), g(\cdot)$ and coefficients m, M, one must verify that A is invertible for all values of the state and choice variables s_t^i, S_t, D_t, and x_t^i.

For the laws of motion eqs. (8) and (9), the matrix A above is:

$$\begin{bmatrix} 1-l^i(\cdot)-s^i_t\frac{\partial l^i}{\partial s^i_t} & 1-l^i(\cdot)-s^i_t\frac{\partial l^i}{\partial s^i_t}-\sum_{j=1}^N s^j_t\frac{\partial l^j}{\partial S_t} & M\left[\frac{\partial l^i}{\partial s^i_t}+\sum_{j=1}^N\frac{\partial l^i}{\partial S_t}\right] \\ -s^i_t\frac{\partial l^i}{\partial S_t} & -\sum_{j=1}^N s^j_t\frac{\partial l^j}{\partial S_t} & M\sum_{j=1}^N\frac{\partial l^i}{\partial S_t} \\ -s^i_t\frac{\partial l^i}{\partial D_t} & -\sum_{j=1}^N s^j_t\frac{\partial l^j}{\partial D_t} & (1-\delta)+M\sum_{j=1}^N\frac{\partial l^i}{\partial D_t}+\frac{\partial g}{\partial D_t} \end{bmatrix} \tag{25}$$

2.3 Social Planner's Program

The social planner (or fleet planner to use Rao and Rondina's terminology), is tasked with maximizing the sum of the operators' benefits $W(\{s^i_t\}, S_t, D_t) = \sum_{i=1}^N V^i(s^i_t, S_t, D_t)$.

$$W(\{s^i_t\}, D_t) = \max_{\{x^i_t\}_{i=1}^N \geq 0} \left(\sum_{i=1}^N u^i(s^i_t, S_t, D_t)\right) - FX_t + \beta W(\{s^i_{t+1}\}, D_{t+1}) \tag{26}$$

Subject To:

$$D_{t+1} = (1-\delta)D_t + mX_t + M\cdot\left(\sum_{i=1}^N s^i_t l^i(s^i_t, S_t, D_t)\right) + g(D_t) \tag{27}$$

$$s^i_{t+1} = \left[1 - l^i(s^i_t, S_t, D_t)\right] s^i_t + x^i_t \tag{28}$$

$$S_t = \sum_{i=1}^N s^i_t \tag{29}$$

$$X_t = \sum_{i=1}^N x^i_t \tag{30}$$

Solving for the euler equation follows the steps laid out in appendix section **??** for constellation operators.

The $N+1$ Envelope Conditions are:

$$\frac{\partial W}{\partial s^i_t} = \sum_{j=1}^N \frac{du^j}{ds^i_t} + \beta\left[\sum_{j=1}^N \frac{\partial W}{\partial s^j_{t+1}}\frac{\partial s^j_{t+1}}{\partial s^i_t} + \frac{\partial W}{\partial D_{t+1}}\frac{\partial D_{t+1}}{\partial s^i_t}\right] \quad \forall i \in \{1,\ldots,N\} \tag{31}$$

$$\frac{\partial W}{\partial D_t} = \sum_{j=1}^N \frac{du^j}{dD_t} + \beta\left[\sum_{j=1}^N \frac{\partial W}{\partial s^j_{t+1}}\frac{\partial s^j_{t+1}}{\partial D_t} + \frac{\partial W}{\partial D_{t+1}}\frac{\partial D_{t+1}}{\partial D_t}\right] \tag{32}$$

$$\nabla W_t - \sum_{j=1}^N \nabla u^j_t = \beta B\cdot\nabla W_{t+1} \tag{33}$$

Assuming B is non-singular, we again find that:

$$\nabla W_{t+1} = (\beta B)^{-1}(\nabla W_t - \sum_{j=1}^N \nabla u^j_t) \tag{34}$$

The N Optimality Conditions are:

$$0 = - F + \beta \left[\sum_{j=1}^{N} \frac{\partial W}{\partial s_{t+1}^{j}} \frac{\partial s_{t+1}^{j}}{\partial x_t^i} + \frac{\partial W}{\partial D_{t+1}} \frac{\partial D_{t+1}}{\partial x_t^i} \right] \quad \forall i \in \{1, \dots, N\} \tag{35}$$

$$\frac{F}{\beta} \vec{1} = C \nabla W_{t+1} \tag{36}$$

Where C is a $N \times N + 1$ matrix. Iterating eq. (35) one period forward (from $t+1$ to $t+2$) for $i = 1$ and and substituting in eq. (34) twice provides the final equation for a system of $N + 1$ equations for ∇W_t.

Finally, iterating eq. (35) one period backward (from $t + 1$ to t) for all i, and substituting the previously found values for ∇W_t into these optimality conditions defines the system of euler equations that characterize $\{x_t^i\}$.

3 Analysis

3.1 Survival Ratios

In line with theory on common-pool resources, we expect there to be a negative externality incurred by increasing the satellite stock. Some details of this externality can be observed in the marginal survival rate. Define the survival rate for a constellation and the society to be:

$$R_i = \frac{s_{t+1}^i - x_t^i}{s_t^i} = 1 - l^i(s_t^i, S_t, D_t) \tag{37}$$

$$R = \frac{S_{t+1} - X_t}{S_t} = \frac{\sum_{i=1}^{N} s_t^i [1 - l^i(s_t^i, S_t, D_t)]}{S_t} \tag{38}$$

The marginal survival rates when a given constellation i changes size are:

$$\frac{\partial R_i}{\partial s_t^i} = - \left(\frac{\partial l^i}{\partial s_t^i} + \frac{\partial l^i}{\partial S_t} \frac{\partial S_t}{\partial s_t^i} \right) = - \frac{\partial l^i}{\partial s_t^i} - \frac{\partial l^i}{\partial S_t} \tag{39}$$

$$\frac{\partial R}{\partial s_t^i} = \frac{S_t \sum_{i=1}^{N} \left([1 - l^i(s_t^i, S_t, D_t)] + s_t^i [- \frac{\partial l^i}{\partial s_t^i} - \frac{\partial l^i}{\partial S_t} \frac{\partial S_t}{\partial s_t^i}] \right) - \left(\sum_{i=1}^{N} s_t^i [1 - l^i(s_t^i, S_t, D_t)] \right)}{(S_t)^2} \tag{40}$$

$$= \sum_{i=1}^{N} \left[\frac{R_i}{S_t} \right] - \frac{R}{S_t} + \sum_{i=1}^{N} \frac{s_t^i}{S_t} \frac{\partial R_i}{\partial s_t^i} \tag{41}$$

Note that $\sum_{i=1}^{N} \frac{s_t^i}{S_t} \frac{\partial R_i}{\partial s_t^i}$ is the average marginal survival rate across constellation operators. The derivation of equation 41 is in Appendix A.1. Direct comparison between the marginal survival rates of an individual operator and the social planner's fleet cannot proceed further without specifying the functional loss forms $l^i(\cdot)$ and specifying which firm will be compared to society. In spite of this, conditions on the average effects can be specified as follows. Society's marginal survival rate is greater than the weighted, arithmetic mean of marginal survival rates of the constellation when:

$$\sum_{i=1}^{N}\left[\frac{R_i}{S_t}\right] - \frac{R}{S_t} + \sum_{i=1}^{N}\frac{s_t^i}{S_t}\frac{\partial R_i}{\partial s_t^i} \leq \sum_{i=1}^{N}\frac{s_t^i}{S_t}\frac{\partial R_i}{\partial s_t^i} \tag{42}$$

$$\sum_{i=1}^{N} R_i - R \leq 0 \tag{43}$$

$$\sum_{i=1}^{N}[1 - l^i(s_t^i, S_t, D_t)] - \sum_{i=1}^{N} s_t^i[1 - l^i(s_t^i, S_t, D_t)] \leq 0 \tag{44}$$

$$\sum_{i=1}^{N}(1 - s_t^i)[1 - l^i(s_t^i, S_t, D_t)] \leq 0 \tag{45}$$

which is true if every constellation has at least one satellite. Based on the definition of constellation survival rate, $s_t^i = 0 \Rightarrow R_i = \frac{0}{0}$ i.e. the survival rate is undefined. In combination with the physical reality that there cannot be a negative number of satellites in a constellation, we are left to conclude that a meaningful constellation has at least one satellite.

As $\frac{\partial R_i}{\partial s_t^i} < 0$ from the assumptions on collision mechanics, we can interpret this result as that the marginal survival rate of the entire satellite fleet is lower than the weighted arithmetic mean of marginal survival rates across constellations. This demonstrates the negative externality of satellite operation, and is a very general condition, consistent with other orbital pollution models. Note that it does allow for some constellations to have a lower marginal survival rate than the fleet, but it can be true as a general condition.

3.2 Kessler Syndrome

Rao and Rondina [7] interpret their model in terms of a physical kessler syndrome, while Adilove et al [12] develop the concept of an economic kessler syndrome. Generalizing Rao's approach, we define the kessler region as the set of states such that the debris stock will tend to infinity, and kessler syndrome as when the state is in the kessler region. Formally, the kessler region is:

$$\vartheta_1 = \left\{(\{s_t^i\}, D_t) : X_t(\{s_t^i\}, D_t) \wedge (\{s_t^i\}, D_t) \Rightarrow \lim_{t\to\infty} D_{t+1} = \infty\right\} \tag{46}$$

I suspect, but have not been able to prove, that an equivalent condition is:

$$\vartheta_2 = \left\{(\{s_t^i\}, D_t) : X_t(\{s_t^i\}, D_t) \wedge (\{s_t^i\}, D_t) \Rightarrow \frac{\partial(D_{t+1} - D_t)}{\partial D_t} > 0\right\} \tag{47}$$

If the assumption holds, then a condition for a physical kessler region in this model is:

$$\vartheta_2 = \left\{ (\{s_t^i\}, D_t) : X_t(\{s_t^i\}, D_t) \wedge (\{s_t^i\}, D_t) \Rightarrow -\delta + m\frac{\partial X_t(\{s_t^i\}, D_t)}{\partial D_t} + M \cdot \left(\sum_{i=1}^{N} \frac{\partial l^i}{\partial D_t} \right) + g(D_t) > 0 \right\} \tag{48}$$

Adilov defines an economic kessler syndrome (and thus kessler region) along the lines of

$$\vartheta_3 = \left\{ (\{s_t^i\}, D_t) : X_t(\{s_t^i\}, D_t) = 0 \right\} \tag{49}$$

This represents the conditions under which adding satellites to the orbit becomes unprofitable. He establishes general conditions under which an economic kessler syndrom precedes a physical kessler syndrome. The benefit of this definition is that the euler equation defining $X_t(\cdot)$ can be searched for the states that imply $X_t = 0, \forall t$ [1].

4 Concluding Remarks

The dynamic model developed in this paper provides insight into the incentives faced by constellation operators in comparison with a social planner and, when completed, should provide insight on how self-perpetuating externalities drive sub-optimal behavior.

At this point, major work remains in identifying optimal launch rates and verifying if the expected difference in optimal launch rates between individual operators and a social planner exist, as occurs in other models. In addition to the remaining work on fleshing out the model, work on the following extensions and applications of the model can fill gaps in the literature or complement current work. Notable areas of interest for future research include:

- Asymmetric constellation sizes: What are the impacts on social welfare when a variety of constellation sizes exist?
- Policy interventions: Various policy proposals to reduce negative externalities have been proposed, including launch quotas, launch taxes, and orbit use fees [**RaoRondina2020b**].
- Strategic behavior: Concerns include whether constellation network effects can be used to prevent new entrants in the case of competition for a satellite services market.

While computationally complicated, the results so far imply that there is a defined difference between the risks faced at the constellation operator's level and the level of society as a whole. Although not a common topic in economics, orbit use has properties that requires current study in order to identify optimal behavior, inform policies, and prevent kessler syndrome before there are no more viable orbits to use.

[1] I have yet to conduct such a search, but plan on doing so as part of a numerical simulation.

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A Derivations

A.1 Survival Rates

$$\frac{\partial R_i}{\partial s_t^i} = -\left(\frac{\partial l^i}{\partial s_t^i} + \frac{\partial l^i}{\partial S_t}\frac{\partial S_t}{\partial s_t^i}\right) = -\frac{\partial l^i}{\partial s_t^i} - \frac{\partial l^i}{\partial S_t} \tag{50}$$

$$\frac{\partial R}{\partial s_t^i} = \frac{S_t \sum_{i=1}^N \left([1 - l^i(s_t^i, S_t, D_t)] + s_t^i[-\frac{\partial l^i}{\partial s_t^i} - \frac{\partial l^i}{\partial S_t}\frac{\partial S_t}{\partial s_t^i}]\right) - \left(\sum_{i=1}^N s_t^i[1 - l^i(s_t^i, S_t, D_t)]\right)}{(S_t)^2} \tag{51}$$

$$= \sum_{i=1}^N \left[\frac{S_t[1 - l^i(s_t^i, S_t, D_t)]}{(S_t)^2} - \frac{s_t^i[1 - l^i(s_t^i, S_t, D_t)]}{(S_t)^2}\right] + \sum_{i=1}^N \frac{s_t^i S_t[-\frac{\partial l^i}{\partial s_t^i} - \frac{\partial l^i}{\partial S_t}]}{(S_t)^2} \tag{52}$$

$$= \sum_{i=1}^N \left[\frac{S_t - s_t^i}{(S_t)^2}[1 - l^i(s_t^i, S_t, D_t)]\right] + \sum_{i=1}^N \frac{s_t^i}{S_t}\frac{\partial R_i}{\partial s_t^i} \tag{53}$$

$$= \sum_{i=1}^N \left[\frac{1}{S_t}[1 - l^i(s_t^i, S_t, D_t)]\right] - \frac{R}{S_t} + \sum_{i=1}^N \frac{s_t^i}{S_t}\frac{\partial R_i}{\partial s_t^i} \tag{54}$$

$$= \sum_{i=1}^N \left[\frac{R_i}{S_t}\right] - \frac{R}{S_t} + \sum_{i=1}^N \frac{s_t^i}{S_t}\frac{\partial R_i}{\partial s_t^i} \tag{55}$$