Dynamic Launch Decisions for Satellite Constellation Operators

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Abstract

Over the last 10 years new technology has make low earth orbits (LEOs) more accessible, and the resulting increase in LEO satellites has increased the risk of collision. Because debris in orbit generates more debris through collisions with objects in orbit and the debris created during launch and operation imposes a negative externality on other operators, optimal use of orbits is believed to not occur under free entry. This paper develops a dynamic model of satellite operation incorporating two effects not considered in previous models. The first effect is complementarity between satellites within the same operator's fleet (called a constellation). The second effect is collision avoidance efficiencies that exist within constellations. The primary result is a theoretical model and the resulting analysis of the difference in survival ratios between constellation operators and society.

Keywords: Orbits, Pollution, Economies of Scale, Externality

JEL Codes: Q29, Q58, L25

1 Introduction

In September of 2019, the European Space Agency (ESA) released a tweet explaining that they had performed an maneuver to avoid a collision with a SpaceX Starlink Satellite in Low Earth Orbit (LEO)[1]. While later reports[2] described it as the result of miscommunications, ESA used the opportunity to highlight the difficulties arising from coordinating avoidance maneuvers and how such coordination will become more difficult as the size and number of single purpose, single operator satellite fleets (satellite constellations) increase in low earth orbit[3].

In spite of the fact that there is a lot of maneuvering room in outer space, the repeated interactions of periodic orbits make collisions probable. Consequently, objects in orbit are subject to both a congestion effect and a pollution effect. Congestion effects are primarily derived from avoiding collisions between artificial satellites. Pollution in orbit consists of debris, both natural and man-made, which increases the probability of an unforeseen collision. The defining dynamic of pollution in orbit is that it self-propagates as debris collides with itself and orbiting satellites to generate more debris. This dynamic underlies a key concern, originally explored by Kessler and Cour-Palais [4] that with sufficient mass in orbit (through satellite launches), the debris generating process could undergo a runaway effect rendering various orbital regions unusable. This cascade of collisions is often known as Kessler syndrome and theoretically may take place over various timescales.

Orbits may be divided into three primary groups, Low Earth Orbit (LEO, less than 2,400km in altitude[5]), Medium Earth Orbit (MEO), and High Earth Orbit (HEO) with Geostationary Earth Orbit (GEO) considered a particular classification of orbit. While the topic of LEO allocation has historically remained somewhat unexplored, the last 6 years has seen a variety of new empirical studies and theoretical models published. In general, three primary, related topics appear in the literature: Allocative Efficiency, Externality Mitigation, and Economic vs Physical Kessler Syndromes.

Although Kessler and Cour-Palais determined that a runaway pollution effect could make a set of orbits physically unusable, Adilov et al [6] have shown that economic benefits provided by orbits will drop sufficiently to make the net marginal benefit of new launches negative before the physical kessler syndrome occurs.

The primary concern is to establish whether or not orbits will be overused due to their common-pool nature, and if allocation procedures are efficient. The earliest theoretical model I have found, due to Adilov, Alexander, and Cunningham [7], examines pollution using a two-period salop model, incorporating the effects of launch debris on survival into the second period. They find that the social planner generates debris and launches at lower rates than a free entry market. This same result was found by Rao and Rondina [8] in the context of an infinite period dynamic model. They approach the problem in the case where numerous operators in a free entry environment can each launch a single, identical satellite.

In addition to analyzing the allocative results, a significant area of interest is what impact various policy interventions can have. The policies analyzed and methods used have been widely varied. Macauley [9] provided the first evidence of sub-optimal behavior in orbit by estimating the welfare lose due to the current method of assigning GEO slots to operators. The potential losses due to anti-competitive behavior was highlighted by Adilov et al [10], who have analyzed the opportunities for strategic “warehousing” of non-functional satellites as a means of increasing competitive advantage by denying operating locations to competitors in GEO. Grzelka and Wagner [11] explore methods of encouraging satellite quality (in terms of debris) and cleanup. Finally, Rao and Rondina [12] estimate that achieving socially optimal behavior through orbital use fees could increase the value generated by the space industry by a factor of 4.

This paper's objective is to develop a dynamic model which incorporates complementary effects of constellations as well as collision avoidance efficiencies of constellations, thus addressing a gap in the current literature. In addition, I examine if there exists a negative externality related to changes in stock size, and establish a condition related to average behavior that describes this externality. Finally, I lay foundations for the derivation of profit maximizing launch rules.

The paper is organized as follows. Section 2 describes the mathematical organization of the model for the cases of independent constellation operators and a social planner operating the same constellations. Section 3 evaluates the differences between the constellation operators and social planner models, particularly the difference between marginal survival rates . Section 4 concludes with a discussion of potential extensions and topics which have not yet been addressed.

2 Model

The dynamic model is an extension of Rao and Rondina's working paper [8], specifically their non-stochastic model. For a given orbital shell (a set of orbits that interact regularly), I assume there are N operators, each of which has the potential to launch and operate a satellite constellation consisting of some endogenously chosen number of identical satellites. These satellites are not only identical within a constellation, but across constellations.

Each constellation operator has a personal satellite stock s_t^i in each period, and chooses the number of launches in that time period x_t^i. For simplicity, each launch is assumed to have a fixed cost F. In the aggregate, the satellite stock and launches for each period are represented by:

$$S_t = \sum_{i=1}^{N} s_t^i \tag{1}$$

$$X_t = \sum_{i=1}^{N} x_t^i \tag{2}$$

Satellites in a constellation are damaged or destroyed at the rate $l^i(s_t^i, S_t, D_t)$, which is assumed to be increasing in s_t^i, S_t, and D_t (debris, see below). One key difference from the previous models of Rao and Rondina [8] and Adilov et al [6] is that this model allows the rate of collision within constellations and between constellations to be different. This reflects the assumption that an operator can and will put more effort into protecting the satellites within the constellation from each other. One example of how this can be accomplished is that while choosing the orbits for a constellation, it is possible for an operator to chose a set of trajectories that best meet their needs and minimizes the risk of collision within the constellation. Mathematically this is represented by the inclusion of s_t^i in l^i. Together with the launch rate, we obtain a law of motion for both constellation-level and society-level satellite stocks.

$$s_{t+1}^i = \left[1 - l^i(s_t^i, S_t, D_t)\right] s_t^i + x_t^i \tag{3}$$

$$S_{t+1} = X_t + \sum_{i=1}^{N} \left[1 - l^i(s_t^i, S_t, D_t)\right] s_t^i \tag{4}$$

The level of debris in each period is represented by D_t, and is assumed to pose a latent risk. In particular, it is assumed that once debris is created, the risk it provides is only avoidable through not launching future satellites. In addition to naturally occurring debris, debris is generated through the following three mechanisms.

- At launch, various processes can shed debris. Examples include leftover rocket stages, explosions during launch and deployment, and slag from solid rocket boosters.
- When destroyed, satellites will fragment and produce debris.
- Debris can collide with other debris, forming more but smaller debris.

This provides the following law of debris dynamics.

$$D_{t+1} = (1-\delta)D_t + mX_t + M \cdot \left(\sum_{i=1}^{N} l^i(s_t^i, S_t, D_t) \right) + g(D_t) \tag{5}$$

where δ represents the decay of debris – through reentering the atmosphere – for a given shell, M represents the debris generated from each collision, m represents the debris generated from each launch, and $g(D_t)$ represents the new fragments from debris colliding with other debris.

Each constellation $i \in 1, \ldots, N$ produces value for their operator at each period according to the function:

$$u^i(s_t^i, S_t, D_t) = u^i(s_t^i) \tag{6}$$

For computational simplicity, it is assumed that benefits provided are wholly dependent on the number of satellites in operation. The approach presented in the appendix is generalizable to the case where benefits are conditional on the total satellite and debris stocks. Complementarity within a constellation appears when $\frac{\partial^2 u^i}{\partial {s_t^i}^2} > 0$ for some values of s_t^i, S_t, D_t.

2.1 Constellation Operator's Program

The aforementioned aspects of the model form the following bellman equation for each constellation operator.

$$V^i(s_t^i, S_t, D_t) = \max_{x_t^i \geq 0} \quad u^i(s_t^i) - Fx_t^i + \beta V^i(s_{t+1}^i, S_{t+1}, D_{t+1}) \tag{7}$$

Subject To:

$$D_{t+1} =(1-\delta)D_t + mX_t + M \cdot \left(\sum_{i=1}^{N} l^i(s_t^i, S_t, D_t) \right) + g(D_t) \tag{8}$$

$$s_{t+1}^i = \left[1 - l^i(s_t^i, S_t, D_t)\right] s_t^i + x_t^i \tag{9}$$

$$S_t = \sum_{i=1}^{N} s_t^i \tag{10}$$

$$X_t = \sum_{i=1}^{N} x_t^i \tag{11}$$

The system of envelope conditions is linear and can be written as a matrix equation. In Appendix A.1 I begin development of the euler equation in a generalizable way. Unfortunately repeated errors in the mathematics has prevented me from achieving more in the launch rate analysis to date.

2.2 Social Planner's Program

The social planner (or fleet planner to use Rao and Rondina's terminology), is tasked with maximizing the sum of the operators' benefits $W(\{s_t^i\}, S_t, D_t) = \sum_{i=1}^{N} V^i(s_t^i, S_t, D_t)$. Often, in polluting environments, there is an ambient population that is harmed by pollution. Very rarely does satellite debris pose a hazard to those on earth, thus in this model the only population who's welfare is addressed are the satellite operators themselves.

$$W(\{s_t^i\}, S_t, D_t) = \max_{\{x_t^i\}_{i=1}^N \geq 0} \left(\sum_{i=1}^N u^i(s_t^i, S_t, D_t) \right) - FX_t + \beta W(\{s_{t+1}^i\}, S_{t+1}, D_{t+1}) \tag{12}$$

Subject To:

$$D_{t+1} = (1-\delta)D_t + mX_t + M \cdot \left(\sum_{i=1}^N l^i(s_t^i, S_t, D_t) \right) + g(D_t) \tag{13}$$

$$s_{t+1}^i = \left[1 - l^i(s_t^i, S_t, D_t)\right] s_t^i + x_t^i \tag{14}$$

$$S_t = \sum_{i=1}^N s_t^i \tag{15}$$

$$X_t = \sum_{i=1}^N x_t^i \tag{16}$$

Due to the aforementioned errors, I have not begun a derivation of the optimal launch rate for the social planner at this point. I expect it to be solvable using the same approach as for the constellation operators outlined in Appendix A.1.

3 Comparisons

In line with theory on common-pool resources, we expect there to be a negative externality incurred by increasing the satellite stock. The details of this externality can be observed in the marginal survival rate. Define the survival rate for a constellation and the society to be:

$$R_i = \frac{s_{t+1}^i - x_t^i}{s_t^i} = 1 - l^i(s_t^i, S_t, D_t) \tag{17}$$

$$R = \frac{S_{t+1} - X_t}{S_t} = \frac{\sum_{i=1}^N s_t^i[1 - l^i(s_t^i, S_t, D_t)]}{S_t} \tag{18}$$

The marginal survival rates when a given constellation i changes size are:

$$\frac{\partial R_i}{\partial s_t^i} = -\left(\frac{\partial l^i}{\partial s_t^i} + \frac{\partial l^i}{\partial S_t}\frac{\partial S_t}{\partial s_t^i} \right) = -\frac{\partial l^i}{\partial s_t^i} - \frac{\partial l^i}{\partial S_t} \tag{19}$$

$$\frac{\partial R}{\partial s_t^i} = \frac{S_t \sum_{i=1}^N \left([1 - l^i(s_t^i, S_t, D_t)] + s_t^i[-\frac{\partial l^i}{\partial s_t^i} - \frac{\partial l^i}{\partial S_t}\frac{\partial S_t}{\partial s_t^i}] \right) - \left(\sum_{i=1}^N s_t^i[1 - l^i(s_t^i, S_t, D_t)] \right)}{(S_t)^2} \tag{20}$$

$$= \sum_{i=1}^N \left[\frac{R_i}{S_t} \right] - \frac{R}{S_t} + \sum_{i=1}^N \frac{s_t^i}{S_t} \frac{\partial R_i}{\partial s_t^i} \tag{21}$$

Note that $\sum_{i=1}^N \frac{s_t^i}{S_t}\frac{\partial R_i}{\partial s_t^i}$ is the average marginal survival rate across constellation operators. The derivation of equation 21 is in Appendix A.3. Direct comparison between the marginal survival rates of an individual operator and the social planner's fleet cannot proceed further without specifying the functional loss forms $l^i(\cdot)$ and specifying which firm the comparison is with. In spite of this, conditions on the average effects can be specified as follows. Society's marginal survival rate is greater than the average marginal survival rate when:

$$\sum_{i=1}^{N}\left[\frac{R_i}{S_t}\right] - \frac{R}{S_t} + \sum_{i=1}^{N}\frac{s_t^i}{S_t}\frac{\partial R_i}{\partial s_t^i} \geq \sum_{i=1}^{N}\frac{s_t^i}{S_t}\frac{\partial R_i}{\partial s_t^i} \tag{22}$$

$$\sum_{i=1}^{N} R_i - R \geq 0 \tag{23}$$

$$\sum_{i=1}^{N} S_t[1 - l^i(s_t^i, S_t, D_t)] - \sum_{i=1}^{N} s_t^i[1 - l^i(s_t^i, S_t, D_t)] \geq 0 \tag{24}$$

$$\sum_{i=1}^{N}(S_t - s_t^i)[1 - l^i(s_t^i, S_t, D_t)] \geq 0 \tag{25}$$

Which is always true as $S_t > s_t^i$ and $l^i(\cdot) \in [0, 1]$ for all i. If a single constellation makes up the whole stock of satellites, then Eq. 25 reduces to a tautology. As $\frac{\partial R_i}{\partial s_t^i} < 0$ from Eq. 19 and the assumptions on collision mechanics, we see that the average marginal survival rate acts as a lower bound on the marginal societal survival rate. Assuming that survival rates are not increased by adding another satellite i.e. $\frac{\partial R}{\partial s_t^i} < 0$ then gives us the following bounds on societal rates $\sum_{i=1}^{N} \frac{s_t^i}{S_t}\frac{\partial R_i}{\partial s_t^i} < \frac{\partial R}{\partial s_t^i} < 0$ This suggests that some operators experience marginal changes to their own satellite stocks much more intensely than society as a whole does.

Once optimal launch rates have been determined, an evaluation of the welfare effects of open access policy can be evaluated, in line with much of the current literature.

4 Concluding Remarks

The dynamic model developed in this paper provides insight into the incentives faced by constellation operators in comparison with a social planner and, when completed, should provide insight on how self-perpetuating externalities drive sub-optimal behavior.

At this point, major work remains in developing optimal launch rates and verifying if the expected difference in optimal launch rates between individual operators and a social planner exist, as occurs in other models. In addition to the remaining work on fleshing out the model, the following extensions and applications of the model will fill gaps in the literature or complement current work:

- Asymmetric constellation sizes: What are the impacts on social welfare when a variety of constellation sizes exist
- Policy interventions: Various policy proposals to reduce negative externalities have been proposed, including launch quotas, launch taxes, and orbit use fees [12].

- Introduction of stochastics: There are various ways that stochastics can enter the model, from the scales determining debris generation to the per-period satellite collision rate.
- Differentiation of satellites and launch methods: Different launch methods and satellite features can affect the accumulation of debris.
- Richer satellite lifetimes: the current satellite lifetime of [launch, operate] could be extended to include stages such as development and disposal. In particular, a multi-period development cycle with sunk costs incurred along the way may exacerbate problems where stable equilibria are overshot. This will allow for more policy interventions to be analyzed.
- Strategic behavior: Concerns include whether constellation network effects can be used to prevent new entrants in the case of competition for a satellite services market.

While computationally complicated, the results so far imply that there is a defined difference between the risks faced at the constellation operator's level and the level of society as a whole. Although not a common topic in economics, orbit use has properties that requires current study in order to determine and drive optimal behavior, before there are no more viable orbits to use.

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A Derivations

A.1 Constellation Operator

Given the following bellman equation

$$V^i(s_t^i, S_t, D_t) = \max_{x_t^i \geq 0} \quad u^i(s_t^i, S_t, D_t) - F x_t^i + \beta V^i(s_{t+1}^i, S_{t+1}, D_{t+1}) \tag{26}$$

Subject To:

$$\begin{aligned} D_{t+1} =& (1-\delta)D_t + mX_t + M \cdot \left(\sum_{i=1}^{N} l^i(s_t^i, S_t, D_t) \right) + g(D_t) && (27) \\ s_{t+1}^i =& \left[1 - l^i(s_t^i, S_t, D_t)\right] s_t^i + x_t^i && (28) \\ S_t =& \sum_{i=1}^{N} s_t^i && (29) \\ X_t =& \sum_{i=1}^{N} x_t^i && (30) \end{aligned}$$

Giving the optimality condition:

$$\frac{F}{\beta} = 2\frac{\partial V^i}{\partial S_{t+1}} + m\frac{\partial V^i}{\partial D_{t+1}} + \frac{\partial V^i}{\partial s_{t+1}^i} \tag{31}$$

Assuming $\frac{\partial u^i}{\partial S_t} = 0$ and $\frac{\partial u^i}{\partial D_t} = 0$, then the envelope conditions are:

$$\begin{aligned} \frac{\partial V^i}{\partial s_t^i} - \frac{\partial u^i}{\partial s_t^i} =& \beta \left[\frac{\partial V^i}{\partial s_{t+1}^i}\frac{\partial s_{t+1}^i}{\partial s_t^i} + \frac{\partial V^i}{\partial S_{t+1}}\frac{\partial S_{t+1}}{\partial s_t^i} + \frac{\partial V^i}{\partial D_{t+1}}\frac{\partial D_{t+1}}{\partial s_t^i} \right] && (32) \\ \frac{\partial V^i}{\partial S_t} =& \beta \left[\frac{\partial V^i}{\partial s_{t+1}^i}\frac{\partial s_{t+1}^i}{\partial S_t} + \frac{\partial V^i}{\partial S_{t+1}}\frac{\partial S_{t+1}}{\partial S_t} + \frac{\partial V^i}{\partial D_{t+1}}\frac{\partial D_{t+1}}{\partial S_t} \right] && (33) \\ \frac{\partial V^i}{\partial D_t} =& \beta \left[\frac{\partial V^i}{\partial s_{t+1}^i}\frac{\partial s_{t+1}^i}{\partial D_t} + \frac{\partial V^i}{\partial S_{t+1}}\frac{\partial S_{t+1}}{\partial D_t} + \frac{\partial V^i}{\partial D_{t+1}}\frac{\partial D_{t+1}}{\partial D_t} \right] && (34) \end{aligned}$$

Note the linearity of the equations. This allows us to rewrite the system as the following matrix expression.

$$\beta \begin{bmatrix} \frac{\partial s_{t+1}^i}{\partial s_t^i} & \frac{\partial S_{t+1}}{\partial s_t^i} & \frac{\partial D_{t+1}}{\partial s_t^i} \\ \frac{\partial s_{t+1}^i}{\partial S_t} & \frac{\partial S_{t+1}}{\partial S_t} & \frac{\partial D_{t+1}}{\partial S_t} \\ \frac{\partial s_{t+1}^i}{\partial D_t} & \frac{\partial S_{t+1}}{\partial D_t} & \frac{\partial D_{t+1}}{\partial D_t} \end{bmatrix} \begin{bmatrix} \frac{\partial V^i}{\partial s_{t+1}^i} \\ \frac{\partial V^i}{\partial S_{t+1}} \\ \frac{\partial V^i}{\partial D_{t+1}} \end{bmatrix} = \begin{bmatrix} \frac{\partial V^i}{\partial s_t^i} - \frac{\partial u^i}{\partial s_t^i} \\ \frac{\partial V^i}{\partial S_t} \\ \frac{\partial V^i}{\partial D_t} \end{bmatrix} \tag{35}$$

$$AD_{[s_{t+1}^i, S_{t+1}, D_{t+1}]}V^i = D_{[s_t^i, S_t, D_t]}V^i - b \tag{36}$$

The matrix A above is equivalent to

$$\begin{bmatrix} 1 - l^i(\cdot) - s_t^i \frac{\partial l^i}{\partial s_t^i} & 1 - l^i(\cdot) - s_t^i \frac{\partial l^i}{\partial s_t^i} - \sum_{j=1}^{N} s_t^j \frac{\partial l^j}{\partial S_t} & M\left[\frac{\partial l^i}{\partial s_t^i} + \sum_{j=1}^{N} \frac{\partial l^i}{\partial S_t}\right] \\ -s_t^i \frac{\partial l^i}{\partial S_t} & -\sum_{j=1}^{N} s_t^j \frac{\partial l^j}{\partial S_t} & M\sum_{j=1}^{N} \frac{\partial l^i}{\partial S_t} \\ -s_t^i \frac{\partial l^i}{\partial D_t} & -\sum_{j=1}^{N} s_t^j \frac{\partial l^j}{\partial D_t} & (1-\delta) + M\sum_{j=1}^{N} \frac{\partial l^i}{\partial D_t} + \frac{\partial g}{\partial D_t} \end{bmatrix} \tag{37}$$

Solving this directly is difficult. We can use the fact that $A^{-1} = \frac{\text{adj}(A)}{\det A}$, assuming A is invertible.

$$D_{[s^i_{t+1}, S_{t+1}, D_{t+1}]} V^i = \frac{\text{adj}(A)}{\beta \det(A)} (D_{[s^i_t, S_t, D_t]} V^i - b) \tag{38}$$

Using each entry from $D_{[s^i_{t+1}, S_{t+1}, D_{t+1}]} V^i$ in the optimality condition and the notation $B|_{i,j}$ to represent the element $b_{i,j}$ from the matrix B, we get the condition:

$$\begin{aligned} \frac{F}{\beta} \beta \det(A) = F \det(A) = & [\text{adj}(A)(D_{[s^i_t, S_t, D_t]} V^i - b)]\big|_1 \\ & + 2[\text{adj}(A)(D_{[s^i_t, S_t, D_t]} V^i - b)]\big|_2 \\ & + m[\text{adj}(A)(D_{[s^i_t, S_t, D_t]} V^i - b)]\big|_3 \end{aligned} \tag{39}$$

A little work remains to develop the euler equation that characterizes the optimal launch decision.

Of course, for any given set of functional forms l^i, g, one must verify if A is invertible.

A.2 Fleet Planner

$$W(\{s^i_t\}, S_t, D_t) = \max_{\{x^i_t\}_{i=1}^N \geq 0} \left(\sum_{i=1}^N u^i(s^i_t, S_t, D_t) \right) - F X_t + \beta W(\{s^i_{t+1}\}, S_{t+1}, D_{t+1}) \tag{40}$$

Subject To:

$$D_{t+1} = (1-\delta) D_t + m X_t + M \cdot \left(\sum_{i=1}^N l^i(s^i_t, S_t, D_t) \right) + g(D_t) \tag{41}$$

$$s^i_{t+1} = \left[1 - l^i(s^i_t, S_t, D_t)\right] s^i_t + x^i_t \tag{42}$$

$$S_t = \sum_{i=1}^N s^i_t \tag{43}$$

$$X_t = \sum_{i=1}^N x^i_t \tag{44}$$

This is expected to follow the constellation operator's results closely.

A.3 Survival Rates

$$\frac{\partial R_i}{\partial s_t^i} = -\left(\frac{\partial l^i}{\partial s_t^i} + \frac{\partial l^i}{\partial S_t}\frac{\partial S_t}{\partial s_t^i}\right) = -\frac{\partial l^i}{\partial s_t^i} - \frac{\partial l^i}{\partial S_t} \tag{45}$$

$$\frac{\partial R}{\partial s_t^i} = \frac{S_t \sum_{i=1}^N \left([1 - l^i(s_t^i, S_t, D_t)] + s_t^i[-\frac{\partial l^i}{\partial s_t^i} - \frac{\partial l^i}{\partial S_t}\frac{\partial S_t}{\partial s_t^i}]\right) - \left(\sum_{i=1}^N s_t^i[1 - l^i(s_t^i, S_t, D_t)]\right)}{(S_t)^2} \tag{46}$$

$$= \sum_{i=1}^N \left[\frac{S_t[1 - l^i(s_t^i, S_t, D_t)]}{(S_t)^2} - \frac{s_t^i[1 - l^i(s_t^i, S_t, D_t)]}{(S_t)^2}\right] + \sum_{i=1}^N \frac{s_t^i S_t[-\frac{\partial l^i}{\partial s_t^i} - \frac{\partial l^i}{\partial S_t}]}{(S_t)^2} \tag{47}$$

$$= \sum_{i=1}^N \left[\frac{S_t - s_t^i}{(S_t)^2}[1 - l^i(s_t^i, S_t, D_t)]\right] + \sum_{i=1}^N \frac{s_t^i}{S_t}\frac{\partial R_i}{\partial s_t^i} \tag{48}$$

$$= \sum_{i=1}^N \left[\frac{1}{S_t}[1 - l^i(s_t^i, S_t, D_t)]\right] - \frac{R}{S_t} + \sum_{i=1}^N \frac{s_t^i}{S_t}\frac{\partial R_i}{\partial s_t^i} \tag{49}$$

$$= \sum_{i=1}^N \left[\frac{R_i}{S_t}\right] - \frac{R}{S_t} + \sum_{i=1}^N \frac{s_t^i}{S_t}\frac{\partial R_i}{\partial s_t^i} \tag{50}$$