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Modeling decisions in operating satellite constellations

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Orbital Debris

ISS threatened by debris cloud - Monday Nov 15th, 2021

- Russia conducts an Anti-Satellite Missle Test generating at least 1,500 items of trackable debris
- The Astronauts and Cosmonauts on the ISS entered lockdown, including donning pressure suits.
- The situation is still being monitored although the immediate danger appears to have passed.

Orbital Debris

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Other events involving the ISS highlight the dangers from orbital debris:

- ISS conducts 3 evasive maneuvers to doge debris in 2020- Jerusalem Post
- ISS hit by debris, May 2021- Canadian Space Agency
- ISS dodged debris from 2007 Anti-Sat Missile, Nov 2021- Jerusalem Post (Same as above)

Orbital Debris

Not just an issue for manned space flight.

Orbital Debris Quarterly News - NASA

- In May of 2020, the Satellite SL-23 Zenit Fregat's tank suffered a second breakup event.
- While only 65 large pieces of debris were initially identified, by Feb. 2021 over 325 had been attributed to the breakup.
- Debris was spread in orbits between 500km and 6,000km.

Starlink and recent Anti-Sat test

- Estimated that there will likely be some impact to Starlink operations.
- 1,500 large pieces of debris initially identified.

Why now?

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In recent years two major changes have occured

1. New launch providers: SpaceX, RocketLab, etc have lead to plummeting launch costs
2. CubeSates and other Nano-Satellites.

Goals

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Goals:

- Model the choices facing Satellite Constellation Operators and optimal policy policy response.
- Investigate the effect of various policies on debris pollution

Overview I

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Past Literature

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Key elements of recent literature.

1. Kessler and Cour-Palais, 1978: Raised issue of runaway orbital pollution.
2. Adilov et al., 2015: Described 2 period salop model of interactions.
3. Adilov et al., 2018a, 2018b: Described an infinite period model with symmetric competitive interactions.
4. A. Rao and Rondina, 2020: Describe a symetric infinite period model (first to do so).
5. Rao et al., 2020: Examine the effect of Orbital-Use fees, find it would quadruple long term value produced of the space industry.

Overview

- Mathematical Notation
- Law of motion for debris
- Law of motion for satellite stocks
- Kessler Syndrome
- Markov Decision Problems

Mathematical Notation

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- S_t: The set of constellation satellites stocks.
- s_t^i: The number of satellites (stock) for constellation i.
- D_t: The level of debris.
- X_t: The set of launches.
- x_t^i: The launches from constellation i.

Debris

Law of motion for debris

$$
\begin{aligned}
D_{t+1} = & (1-\delta)D_t && \text{(Debris decay.)} \\
& + g \cdot D_t && \text{(Debris produced by collision with debris.)} \\
& + \gamma \sum_{i=1}^{N} \left(1 - R^i(S_t, D_t)\right) s_t^i && \text{(Debris produced by satellite destruction.)} \\
& + \Gamma \sum_{j=1}^{n} x_t^j && \text{(Debris produced by launches.)}
\end{aligned}
$$

Satellite Stocks

Law of motion for satellite stocks

$$s^i_{t+1} = \left(R^i(S_t, D_t, X_t) - \eta\right) \cdot s^i_t + x^i_t \tag{1}$$

- η is the orbit decay rate of satellites.

Explanation of Kessler Syndrome

Kessler Syndrome

The situation in which collisions between objects in orbit produced debris and this debris begins collisions with other objects, leading to a runaway growth in debris. As debris can persist for millenia, this may make some orbits unusable. (Kessler & Cour-Palais, 1978)

Often described as a condition with an exponential growth of debris.

Past approaches to Kessler Syndrome

- Adilov et al., 2018b:
  Develops an analog of kessler syndrome where the condition is met when satellites are destroyed immediately after launch by debris.

$$\{(S_t, D_t) : R^i(S_t, D_t) = 0 \forall i\} \tag{2}$$

- A. Rao and Rondina, 2020:
  A working paper in which the authors develop a dynamic model and a definition of kessler syndrome that captures all increasing debris levels.

$$\left\{(S_t, D_t) : \lim_{t \to \infty} D_{t+1}(S_t, D_t) = \infty\right\} \tag{3}$$

My contributions

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I propose two refinements of these definitions to simplify analyzing kessler syndrome in computational models.

- ϵ-Kessler Region
- Proto Kessler Region

ϵ-Kessler Region

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ϵ-Kessler Region

$$\kappa_\epsilon = \{(S_t, D_t) : \forall k \geq 0, \quad D_{t+k+1} - D_{t+k} \geq \epsilon > 0\} \tag{4}$$

Notable Features

- ϵ can be calibrated to capture only economically significant growth.
- Requires an explicit description of what is considered economically significant.
- Guarantees divergent behavior.
- Simulated transition paths can identify the region.

Proto Kessler Region

Proto Kessler Region

$$\kappa_{\text{proto}} = \{(S_t, D_t) : \quad D_{t+1} - D_t \geq \epsilon_{\text{proto}}\} \tag{5}$$

Notable Features

- ϵ_{proto} can be calibrated to capture only economically significant growth.
- Requires an explicit description of what is considered economically significant.
- Does not guarantee divergent behavior.
- Easily computable kessler regions.

Proto Kessler Region

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With the given law of motion for debris, the proto-kessler region is:

$$\left\{(S_t, D_t) : (g - \delta)D_t + \gamma \sum_{i=1}^{n} 1 - R^i(S_t, D_t) + \Gamma \sum_{i=1}^{n} x_t^i(S_t, D_t) \geq \epsilon_{\text{proto}}\right\} \quad (6)$$

Operator's Problem

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$$V^i(S_t, x_t^{\sim i}, D_t) = \max_{x_t^i} u^i(S_t, D_t) - F(x_t^i) + \beta \left[V^i(S_{t+1}, x_{t+1}^{\sim i}, D_{t+1}) \right] \qquad (7)$$

Benefit Functions

Possible benefit functions

- Linear (Currently working on this one)
- Cournot Profits
- Profits under Partial substitutability
- Military capabilities (Keeping up with the Jones')

Planner's Problem

$$W(S_t, D_t) = \max_{X_t} \left[\sum_{i=1}^{N} \left(u^i(S_t, D_t) - F(x_t^i) \right) + \beta \left[W(S_{t+1}, D_{t+1}) \right] \right]$$

subject to:

$$s_{t+1}^i = (R^i(S_t, D_t)) s_t^i + x_t^i \quad \forall i$$

$$D_{t+1} = (1 - \delta + g) D_t + \gamma \sum_{i=1}^{N} \left(1 - R^i(\vec{s}_t, D_t) \right) s_t^i + \Gamma \sum_{i=1}^{N} x_t^i \tag{8}$$

Planned model expansions

- Multiple interacting orbital shells and debris terms.
- Stochastic laws of motion
- Multiple types of operators
- Operators benefit functions include taxation

Issues

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The following issues characterize the Operators' and Planner's problem

- Curse of Dimensionality
- Strategic Interaction (operators only)

Possible approaches

Possible approaches

- Standard VFI/Howards algorithm.
- VFI with sparse state space (dimensionality reduction).
- Reinforcement Learning.
- **MALIAR2018** approaches using machine learning.

Chosen approach

Bellman Residual minimization due to (**MALIAR2018**)
Use NN to approximate $V(S_t, D_t|\theta_1)$ and $X(S_t, D_t|\theta_2)$.
The loss function is:

$$0 = [V(S_t, D_t) - F(S_t, D_t, X_t) - \beta V(S_{t+1}, D_{t+1})]^2 - v\,[F(S_t, D_t, X_t) + \beta V(S_{t+1}, D_{t+1})] \tag{9}$$

$$0 = \left[V(S_t, D_t) - F(S_t, D_t, X_t) - \beta V(S_{t+1}, D_{t+1}) - \frac{v}{2}\right]^2 - v\left[V(S_t, D_t) + \frac{v}{4}\right] \tag{10}$$

Training Loop: Planner

For each training epoch

1. Draw random data
2. train policy function
3. train value function

Training Loop: Operators

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For each training epoch

1. Draw random data
2. For each operator
    1. train policy function
    2. train value function

State of the Code

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Currently functioning

- Planner Value and Policy training

Almost functioning

- Operator Value and Policy training
- Proto-Kessler Region analysis

Results

Results are currently waiting on finishing the code.

Some analyses I plan on completing include

- Kessler Region analysis
- Free Entry conditions analysis

Summary

Summary

1. Created Dynamic model of the MDP facing satellite operators.
2. Defined new Kessler Regions for computational analysis.
3. Currently developing solution and simulation tools.
4. Much work left to do.

Other Areas Needing Work

Related Orbits Work

1. Adding stochastic elements to the model.
2. Parameter Estimation.
3. Rights of Way.
4. Satellite Lifetimes and constellation management.

Related computational work

1. Automating the Euler Equation Residuals method.

Questions?

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Any remaining questions?

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Adilov, N., Alexander, P. J., & Cunningham, B. M. (2018a). Corrigendum to "an economic "kessler syndrome": A dynamic model of earth orbit debris" [econom. lett. 166 (2018) 79–82]. *170*, 185. https://doi.org/10.1016/j.econlet.2018.04.012

Adilov, N., Alexander, P. J., & Cunningham, B. M. (2018b). An economic "kessler syndrome": A dynamic model of earth orbit debris. *166*, 79–82. https://doi.org/10.1016/j.econlet.2018.02.025

Adilov, N., Alexander, P. J., & Cunningham, B. M. (2015). Earth orbit debris: An economic model. https://doi.org/10.2139/ssrn.2264915

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Kessler, D. J., & Cour-Palais, B. G. (1978). Collision frequency of artificial satellites: The creation of a debris belt. *Journal of Geophysical Research: Space Physics*, *83*(A6), https://agupubs.onlinelibrary.wiley.com/doi/pdf/10.1029/JA083iA06p02637, 2637–2646. https://doi.org/10.1029/JA083iA06p02637

Rao, A., & Rondina, G. (2020). *Cost in space:debris and collision risk in the orbital commons* (Working Paper) [Middlebury College — UC San Diego]. NA. Middlebury College — UC San Diego.

Rao, Burgess, & Kaffine. (2020). Orbital-use fees could more than quadruple the value of the space industry. *Proceedings of the National Academy of Sciences*, *117*(23), https://www.pnas.org/content/117/23/12756.full.pdf, 12756–12762. https://doi.org/10.1073/pnas.1921260117