Dynamic Launch Decision for Satellite Constellation Operators

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Introduction

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For the first time ever, ESA has performed a 'collision avoidance manoeuvre' to protect one of its satellites from colliding with a 'mega constellation' #SpaceTraffic (Agency, 2019)

In 1978,Donald Kessler and Burton Cour-Palais identified a potential threat to the new frontier of Earth Orbit. They suggested that if there are enough objects in orbit, debris colliding with other debris and artificial satellites could create debris at an increasing rate, leading to an uncontrollable cascade of collisions, now termed kessler syndrome (Kessler & Cour-Palais, 1978).

My goal is to evaluate how the organization of satellite operations into "constellations" affects pollution dynamics and the incentives of operators to deviate from socially optimal behaviors.

Overview

Literature

- (Macauley, 1998) : Estimates the welfare loss due to inefficient allocation of geostationary orbit slots.
- (Adilov et al., 2015) : Two period model evaluating launch decisions.
- (Adilov et al., 2018) : Develop an economic Kessler syndrome where pollution is sufficient to halt launches.
- (Rao & Rondina, 2020) : A widely cited working paper developing the first dynamic model of orbit allocations. Originates in Rao's dissertation from 2015.
- (Adilov et al., 2019) : Develops a dynamic model evaluating competitive interactions between firms.
- (Rao et al., 2020) : Estimates the impact of implementing satellite taxes on future profitability of the satellite industry.

High level description

This model is the first dynamic model to incorporate effects from organization as constellations. These effects enter in two forms:

1. Economies of scale in value production.
2. Collision avoidance efficiencies from constellation planning.

Key features of this model are:

- The assumption that each constellation creates utility without competitive interactions (i.e. monopolistically).
- Each satellite within a constellation is considered identical. Only the number of satellites contributes to the value produced.

These features simplify computation significantly.

Mathematical Terms

Symbol	Details	Description
N	$N > 0$	Number of constellations
s_t^i	$i \in \{1, \ldots, N\}$	Satellite stock of i in t
x_t^i	Ditto	Launches of satellites in t by i
S_t		Total number of satellites in t
D_t	$D_t \geq 0$	Level of debris in t
m, M	$m > 0, M > 0$	Debris generated from launches and collisions respectively
$g(D_t)$		Debris generated from collisions with debris
δ	$\delta \in (0, 1)$	Decay rate of debris
$l^i(s_t^i, S_t, D_t)$	$l^i() \in (0, 1)$	Rate of satellite loss in i due to collisions
$u^i(s_t^i, S_t, D_t)$		Utility generated by satellite stock s_t^i given S_t, D_t.

Constellation Operator's Problem

$$V^i(s_t^i, S_t, D_t) = \max_{x_t^i \geq 0} \quad u^i(s_t^i, S_t, D_t) - Fx_t^i + \beta V^i(s_{t+1}^i, S_{t+1}, D_{t+1}) \tag{1}$$

Subject To:

$$D_{t+1} = (1-\delta)D_t + mX_t + M \cdot \left(\sum_{i=1}^{N} s_t^i l^i(s_t^i, S_t, D_t) \right) + g(D_t) \tag{2}$$

$$s_{t+1}^i = \left[1 - l^i(s_t^i, S_t, D_t)\right] s_t^i + x_t^i \tag{3}$$

$$S_t = \sum_{i=1}^{N} s_t^i \quad X_t = \sum_{i=1}^{N} x_t^i \tag{4}$$

Solving Constellation's Problem I

The general envelope conditions are:

$$\frac{\partial V^i}{\partial s_t^i} - \frac{d u^i}{d s_t^i} = \beta \left[\frac{\partial V^i}{\partial s_{t+1}^i} \frac{\partial s_{t+1}^i}{\partial s_t^i} + \frac{\partial V^i}{\partial S_{t+1}} \frac{\partial S_{t+1}}{\partial s_t^i} + \frac{\partial V^i}{\partial D_{t+1}} \frac{\partial D_{t+1}}{\partial s_t^i} \right] \tag{5}$$

$$\frac{\partial V^i}{\partial S_t} - \frac{d u^i}{d S_t} = \beta \left[\frac{\partial V^i}{\partial s_{t+1}^i} \frac{\partial s_{t+1}^i}{\partial S_t} + \frac{\partial V^i}{\partial S_{t+1}} \frac{\partial S_{t+1}}{\partial S_t} + \frac{\partial V^i}{\partial D_{t+1}} \frac{\partial D_{t+1}}{\partial S_t} \right] \tag{6}$$

$$\frac{\partial V^i}{\partial D_t} - \frac{d u^i}{d D_t} = \beta \left[\frac{\partial V^i}{\partial s_{t+1}^i} \frac{\partial s_{t+1}^i}{\partial D_t} + \frac{\partial V^i}{\partial S_{t+1}} \frac{\partial S_{t+1}}{\partial D_t} + \frac{\partial V^i}{\partial D_{t+1}} \frac{\partial D_{t+1}}{\partial D_t} \right] \tag{7}$$

$$\nabla V_t^i - \nabla u_t^i = \beta A \nabla V_{t+1}^i \tag{8}$$

Solving Constellation's Problem II

The optimality conditions is:

$$\frac{F}{\beta} = \frac{\partial V^i}{\partial s^i_{t+1}} + \frac{\partial V^i}{\partial S_{t+1}} + m\frac{\partial V^i}{\partial D_{t+1}} \tag{9}$$

Iterating both forward and backward one period gives the system

$$\frac{F}{\beta} = \frac{\partial V^i}{\partial s^i_{t}} + \frac{\partial V^i}{\partial S_{t}} + m\frac{\partial V^i}{\partial D_{t}} \tag{10}$$

$$\frac{F}{\beta} = \frac{\partial V^i}{\partial s^i_{t+1}} + \frac{\partial V^i}{\partial S_{t+1}} + m\frac{\partial V^i}{\partial D_{t+1}} \tag{11}$$

$$\frac{F}{\beta} = \frac{\partial V^i}{\partial s^i_{t+2}} + \frac{\partial V^i}{\partial S_{t+2}} + m\frac{\partial V^i}{\partial D_{t+2}} \tag{12}$$

Solving Constellation's Problem III

Thus by iterating eqs. (5) to (7) to match eqs. (10) to (12), we can simplify from 9 equations with 9 unknowns to 3 equations with 3 unknowns allowing us to solve for $\nabla_{[s_t^i, S_t, D_t]} V_t$ in terms of derivatives of the utility function and derivatives of the laws of motion.

Substituting ∇V_t into the equation below (eq. (10)) provides the euler equation that characterizes the policy function $x_t^i(s_T^i, S_t, D_t)$.

$$\frac{F}{\beta} = \frac{\partial V^i}{\partial s_t^i} + \frac{\partial V^i}{\partial S_t} + m\frac{\partial V^i}{\partial D_t}$$

Social Planner's Problem

We can address the social planner's problem in much the same way.

$$W(\{s_t^i\}, D_t) = \max_{\{x_t^i\}_{i=1}^N \geq 0} \left(\sum_{i=1}^N u^i(s_t^i, S_t, D_t) \right) - FX_t + \beta W(\{s_{t+1}^i\}, D_{t+1}) \quad (13)$$

Subject To:

$$D_{t+1} = (1-\delta)D_t + mX_t + M \cdot \left(\sum_{i=1}^N s_t^i l^i(s_t^i, S_t, D_t) \right) + g(D_t) \quad (14)$$

$$s_{t+1}^i = \left[1 - l^i(s_t^i, S_t, D_t)\right] s_t^i + x_t^i \quad (15)$$

$$S_t = \sum_{i=1}^N s_t^i \quad X_t = \sum_{i=1}^N x_t^i \quad (16)$$

Solving Planner's Problem I

The $N+1$ Envelope Conditions are:

$$\frac{\partial W}{\partial s_t^i} = \sum_{j=1}^{N} \frac{d u^j}{d s_t^i} + \beta \left[\sum_{j=1}^{N} \frac{\partial W}{\partial s_{t+1}^j} \frac{\partial s_{t+1}^j}{\partial s_t^i} + \frac{\partial W}{\partial D_{t+1}} \frac{\partial D_{t+1}}{\partial s_t^i} \right] \quad \forall i \in \{1, \dots, N\} \tag{17}$$

$$\frac{\partial W}{\partial D_t} = \sum_{j=1}^{N} \frac{d u^j}{d D_t} + \beta \left[\sum_{j=1}^{N} \frac{\partial W}{\partial s_{t+1}^j} \frac{\partial s_{t+1}^j}{\partial D_t} + \frac{\partial W}{\partial D_{t+1}} \frac{\partial D_{t+1}}{\partial D_t} \right] \tag{18}$$

(19)

Solving Planner's Problem II

The N Optimality Conditions are:

$$0 = -F + \beta \left[\sum_{j=1}^{N} \frac{\partial W}{\partial s_{t+1}^{j}} \frac{\partial s_{t+1}^{j}}{\partial x_t^i} + \frac{\partial W}{\partial D_{t+1}} \frac{\partial D_{t+1}}{\partial x_t^i} \right] \quad \forall i \in \{1, \ldots, N\} \tag{20}$$

Iterating eq. (20) one period forward (from $t+1$ to $t+2$) for $i = 1$ and and substituting in the correctly iterated envelope conditions provides the final equation for a system of $N+1$ optimality conditions for ∇W_t.
Once again, iterating eq. (20) backwards from $t+1$ to t and substituting in ∇W_t will allow you to find the N euler equations characterizing the policy functions $\{x_t^i\}$.

Welfare analysis

A standard result in the models mentioned in the slide on previous work is that of how free entry or competitive use results in launching more than the socially optimal number of satellites. I suspect that result will hold true in this model. *Unfortunately I have not been able to do more than these derivations. The welfare analysis will involve some numerical methods at some point as it gets very messy.*

Survival Rates

One key analysis in Rao and Rondina, 2020 is about the survival rates of satellites. Define the survival rate for a constellation and the society to be:

$$R_i = \frac{s_{t+1}^i - x_t^i}{s_t^i} = 1 - l^i(s_t^i, S_t, D_t) \tag{21}$$

$$R = \frac{S_{t+1} - X_t}{S_t} = \frac{\sum_{i=1}^N s_t^i [1 - l^i(s_t^i, S_t, D_t)]}{S_t} \tag{22}$$

Survival Rates

The marginal survival rates when a given constellation i changes size are:

$$\frac{\partial R_i}{\partial s_t^i} = -\left(\frac{\partial I^i}{\partial s_t^i} + \frac{\partial I^i}{\partial S_t}\frac{\partial S_t}{\partial s_t^i}\right) \leq 0 \tag{23}$$

$$\frac{\partial R}{\partial s_t^i} = \sum_{i=1}^{N}\left[\frac{R_i}{S_t}\right] - \frac{R}{S_t} + \sum_{i=1}^{N}\frac{s_t^i}{S_t}\frac{\partial R_i}{\partial s_t^i} \tag{24}$$

Survival Rates

Thus society's marginal survival rate is less than the weighted arithemetic mean of survival rates for individually growing constellations when:

$$\sum_{i=1}^{N}\left[\frac{R_i}{S_t}\right] - \frac{R}{S_t} + \sum_{i=1}^{N}\frac{s_t^i}{S_t}\frac{\partial R_i}{\partial s_t^i} \leq \sum_{i=1}^{N}\frac{s_t^i}{S_t}\frac{\partial R_i}{\partial s_t^i} \tag{25}$$

$$\sum_{i=1}^{N} R_i - R \leq 0 \tag{26}$$

$$\sum_{i=1}^{N}[1 - l^i(s_t^i, S_t, D_t)] - \sum_{i=1}^{N} s_t^i[1 - l^i(s_t^i, S_t, D_t)] \leq 0 \tag{27}$$

$$\sum_{i=1}^{N}(1 - s_t^i)[1 - l^i(s_t^i, S_t, D_t)] \leq 0 \tag{28}$$

This condition is met as every constellation consists of at least one satellite.

Economic Kessler Syndrome

Adilov et al., 2018 develop a description of economic kessler syndrom as when the debris and satellite stocks are such that it is not profitable to launch.
Mathematically this is:

$$\vartheta_3 = \left\{(\{s_t^i\}, D_t) : X_t(\{s_t^i\}, D_t) = 0\right\} \tag{29}$$

This definition has the benefit that it can be found through a numerical search directly on the euler equations developed previously.
Again, I have not been able to implement this analysis.

Conclusion

Summary: In this paper I have described a model and general set of euler equations describing the decisions facing satellite constellation operators. Additionally I have established that negative pollution externalities exist, consitent with other models. This model provides a basis for analyses of competitive and non-competitive interaction between constellation operators, and for the analysis of policy interventions.

Future Work: There remains significant work to finalize the model, including exploring a numerical model, clarifying existence criteria, and verifying if constellation operators are likely to overuse orbits.

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