In Kessler and Cour-Palais, 1978 the authors described and forecasted what has come to be known as "kessler syndrome", where debris collides with itself in such a way that the overall debris level grows exponentially. A few methods have been used to model this behavior in the economics literature.

The first one I want to explain was developed by Adilov et al., 2018. They characterize kessler syndrome as the point in time at which an orbit is unusable as each satellite launched will be destroyed within a single time period. In my notation, this is that $l^i(\{s^j_t\}, D_t) = 1$. The benefit of this approach is that it is algebraically simple. It was used in this role to show that firms will stop launching before orbits are rendered physically useless. Unfortunately, it does not convey the original intent of "kessler syndrome", i.e. a runaway pollution effect, but instead corresponds to the end result of kessler syndrome.

The second common definition of "kessler syndrome" is due to **RaoRondina**. They define it in terms of a "kessler region", the set of satellite stocks and the debris level such that:

$$\kappa = \left\{ \{s^j_t\}, D_t : \lim_{k\to\infty} D_{t+k}\left(\{s^j_{t+k-1}\}, D_{t+k-1}, \{x^j\}\right) = \infty \right\} \quad (1)$$

0.1 My approach to kessler syndrome

I propose to analyze kessler syndrome in a slightly more restricted fashion than **RaoRondina**. I would define the ϵ-kessler region as:

$$\kappa = \left\{ \{s^j_t\}, D_t : \forall k \geq 0, D_{t+k+1} - D_{t+k} \geq \epsilon > 0 \right\} \quad (2)$$

It is easily shown that this criteria is sufficient to guarantee Rao and Rondina's criteria. It has three primary benefits:

- The ϵ-kessler region can be numerically described within bounded state spaces.
- In a computational model, as most models of any complexity will be, you cannot check for divergence numerically. The condition given is a basic guarantee of the divergent behavior that is required for Kessler Syndrome and acknowledges computational limitations.
- Finally, a slow divergence is no divergence in the grand scheme of things. It is possible to have a mathematically divergent function, but one that is so slow, there is no noticable degree of debris growth before Sol enters a red giant phase. In this specification, it is possible to choose ϵ such that the divergent behaviors identified have an impact on a meaningful timescale.

There is at least one issue with this definition of ϵ-kessler regions. Let's define a "proto-kesslerian" region as the stock and debris levels such that:

$$\kappa = \left\{ \{s^j_t\}, D_t : D_{t+1} - D_t \geq \epsilon > 0 \right\} \quad (3)$$

It may be, under certain situations, the case that optimal launch rates cycle along with debris and stock levels, leading to a cycle in and out of the proto-kesslerian regions. This is an issue because, assumning a stable cycle, Rao's definition of the kessler region would capture this behavior, but the ϵ-kessler definition would not. I believe, but have not verified, that some choices of ϵ, although permitting cycles, would relegate them to unimportant levels.

This leads to the important question of what makes a good value of ϵ? One method, in the spirit of Adilov et al., 2018, is to choose a change in debris, $D_{t+1} - D_t$, such that the loss of satellites in periods $t+1$ to $t+k$ is increased by or to a certain percentage, say 50%. I've put very little thought into addressing this general question so far, and need to analyze the implications of different choice rules.